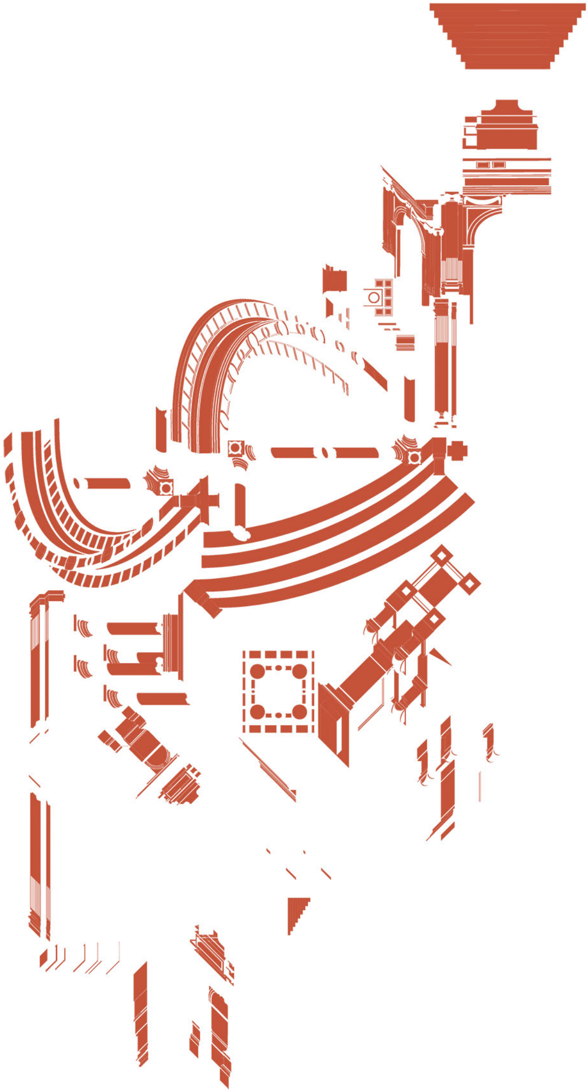

Ways of Drawing

Kamal Ranchod
Hyperreal Perspicuities
Unit 18 2020

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Rhizome

A philosophical concept developed by Gilles Deleuze and Félix Guattari in their Capitalism and Schizophrenia (1972–1980) project. It is what Deleuze calls an "image of thought", based on the botanical rhizome, that apprehends multiplicities.

A Thousand Plateaus by Félix Guattari and Gilles Deleuze:

Plateau, rhizomes are how multiplicities connect. Plateaus are types of intensities held together without beginning or end that develops to a climax.
Subverts linear argumentation and postulation.
Lateral growth without a centre or periphery, it is a-centred, anti-hierarchical, no beginning or end, no middle. An assemblage of multiplicities.
Arborescent models are the contrasting opposite, and they develop a privileged viewpoint.
Arborescent models use an over coding signifier that dominates other regimes.
The is no privileged view in a rhizome, and it doesn't subjectify any other views.
Rhizomes are below the surface.
Two types define assemblage. First is machinic which have to do with material and social flows and assemblages of annunciation which have to with sign regimes, linguistics and language.

Plane of consistency vs plane of organization. Plane of consistency is the consistency of multiplicities that fill their dimensions. Plane of organization becomes the plane of law of hierarchal apparatuses that over codes the plane of consistency.
Three types of novels: Linear, Cyclical, and Rhizomatic.

Six principles of a rhizome are:
1. Connection - any point connects with any other point, there is no privileged centre. Increase in dimensions, multiple points of entry, fragmentation.
2. Heterogeneity - what is connected can be two completely different multiplicities (orchid & Wasp). A-Parallel- Evolution. The phenomenon of deterritorialization and reterritorialization.
3. Multiplicity - Weave together to form vast tapestries, interconnectedness.
4. A-Signifying Rupture - Doesn't have a central failure point such as arborescent models, when rhizomes are ruptured, they can create a line of flight and reorganize itself. Line of flight is central to the idea of deterritorialization and reterritorialization. Deterritorialization and reterritorialization happens when a multiplicity is recoded with a different set of functions.These can be altered from its original functions. Lips originally territorialized for eating but then become de-territorialized and reterritorialized for language. This creates new lines of segmentarity.
5. Cartography - Rhizomes create maps, maps are always unfinished, and you can enter them at any point.
6. Decalcomania - tracing and engraving that is transferred to different surfaces, tracings are complete, using preset categories and apparatuses. Tracings can be placed on the map.

Deleuze, G., Guattari, F. and Massumi, B., 2005. A Thousand Plateaus. Minneapolis: University of Minnesota Press.

Extensivities vs instensivities . Extensivities are phenomena of the physical world that are divisible, measurable and quantitative. Intensivities are those aspects of phenomena that cannot be divided without fundamentally changing the nature of the system. These link with the planes of consistencies.

The Strata are acts of capture which capture flows, continua of intensities, singularities to code and territorialize them.
Each stratum has a double articulation of content and expression. Both contents and expressions consist of forms and substances. The plane of consistencies is the exterior of all multiplicities. A different abstract machine organizes each stratum. The abstract machine is embedded in the stratum to give it a unity of composition. The Canvas is the body without organs; the artist's idea of what is to paint is the abstract machine, the machinic assemblage that effectuates the body without organ into the canvas is the paintbrush.

Macro politics and micropolitics, society is always segmented. There are three different types of segmentation:
1.Binary Type
2.Concentric Type of Segmentation - concentric radiuses with concentric circles of segmentation. My to home, to my suburb to my city etc.
3.Linear Segmentation - Procedural as moving project to project

Deleuze, G., Guattari, F. and Massumi, B., 2005. A Thousand Plateaus. Minneapolis: University of Minnesota Press.

Cartography

Combining science, aesthetics, and technique, cartography builds on the premise that reality can be modeled in ways that communicate spatial information effectively.

The fundamental uses of traditional cartography are to:

Set the map's agenda and select traits of the object to be mapped. This is the concern of map editing. Traits may be physical, such as roads or land masses, or may be abstract, such as toponyms or political boundaries.
Represent the terrain of the mapped object on flat media. This is the concern of map projections.
Eliminate characteristics of the mapped object that are not relevant to the map's purpose. This is the concern of generalization.
Reduce the complexity of the characteristics that will be mapped. This is also the concern of generalization.
Orchestrate the elements of the map to best convey its message to its audience. This is the concern of map design.

Map Projections

Type of projections:

Cylindrical - In standard presentation, these map regularly-spaced meridians to equally spaced vertical lines, and parallels to horizontal lines.
Pseudocylindrical - In standard presentation, these map the central meridian and parallels as straight lines. Other meridians are curves (or possibly straight from pole to equator), regularly spaced along parallels.
Conic - In standard presentation, conic (or conical) projections map meridians as straight lines, and parallels as arcs of circles.
Pseudoconical - In standard presentation, pseudoconical projections represent the central meridian as a straight line, other meridians as complex curves, and parallels as circular arcs.
Azimuthal - In standard presentation, azimuthal projections map meridians as straight lines and parallels as complete, concentric circles. They are radially symmetrical. In any presentation (or aspect), they preserve directions from the center point. This means great circles through the central point are represented by straight lines on the map.
Pseudoazimuthal - In standard presentation, pseudoazimuthal projections map the equator and central meridian to perpendicular, intersecting straight lines. They map parallels to complex curves bowing away from the equator, and meridians to complex curves bowing in toward the central meridian. Listed here after pseudocylindrical as generally similar to them in shape and purpose.

Other:
Typically calculated from formula, and not based on a particular projection
Polyhedral maps - Polyhedral maps can be folded up into a polyhedral approximation to the sphere, using particular projection to map each face with low distortion.
Properties:
Conformal
Preserves angles locally, implying that local shapes are not distorted and that local scale is constant in all directions from any chosen point.
Equal-area
Area measure is conserved everywhere.
Compromise
Neither conformal nor equal-area, but a balance intended to reduce overall distortion.
Equidistant
All distances from one (or two) points are correct. Other equidistant properties are mentioned in the notes.
Gnomonic
All great circles are straight lines.
Retroazimuthal
Direction to a fixed location B (by the shortest route) corresponds to the direction on the map from A to B.

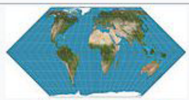
Projection	Image	Type	Properties	Creator	Year	Notes
Equirectangular = equidistant cylindrical = rectangular = la carte parallélogrammatique		Cylindrical	Equidistant	Marinus of Tyre	c. 120	Simplest geometry; distances along meridians are conserved. Plate carrée: special case having the equator as the standard parallel.
Cassini = Cassini–Soldner		Cylindrical	Equidistant	César-François Cassini de Thury	1745	Transverse of equidistant projection; distances along central meridian are conserved. Distances perpendicular to central meridian are preserved.
Mercator = Wright		Cylindrical	Conformal	Gerardus Mercator	1569	Lines of constant bearing (rhumb lines) are straight, aiding navigation. Areas inflate with latitude, becoming so extreme that the map cannot show the poles.
Web Mercator		Cylindrical	Compromise	Google	2005	Variant of Mercator that ignores Earth's ellipticity for fast calculation, and clips latitudes to ~85.05° for square presentation. De facto standard for Web mapping applications.

Cartography

En.wikipedia.org. 2020. List Of Map Projections. [online] Available at: <https://en.wikipedia.org/wiki/List_of_map_projections> [Accessed 14 May 2020].

En.wikipedia.org. 2020. List Of Map Projections. [online] Available at: <https://en.wikipedia.org/wiki/List_of_map_projections> [Accessed 14 May 2020].

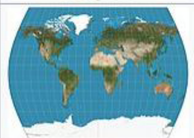
Gauss–Krüger = Gauss conformal = (ellipsoidal) transverse Mercator		Cylindrical	Conformal	Carl Friedrich Gauss Johann Heinrich Louis Krüger	1822	This transverse, ellipsoidal form of the Mercator is finite, unlike the equatorial Mercator. Forms the basis of the Universal Transverse Mercator coordinate system.
Roussilhe oblique stereographic				Henri Roussilhe	1922	
Hotine oblique Mercator		Cylindrical	Conformal	M. Rosenmund, J. Laborde, Martin Hotine	1903	
Gall stereographic		Cylindrical	Compromise	James Gall	1855	Intended to resemble the Mercator while also displaying the poles. Standard parallels at 45°N/S.
Miller = Miller cylindrical		Cylindrical	Compromise	Osborn Maitland Miller	1942	Intended to resemble the Mercator while also displaying the poles.
Lambert cylindrical equal-area		Cylindrical	Equal-area	Johann Heinrich Lambert	1772	Standard parallel at the equator. Aspect ratio of π (3.14). Base projection of the cylindrical equal-area family.
Behrmann		Cylindrical	Equal-area	Walter Behrmann	1910	Horizontally compressed version of the Lambert equal-area. Has standard parallels at 30°N/S and an aspect ratio of 2.36.
Hobo–Dyer		Cylindrical	Equal-area	Mick Dyer	2002	Horizontally compressed version of the Lambert equal-area. Very similar are Trystan Edwards and Smyth equal surface (= Craster rectangular) projections with standard parallels at around 37°N/S. Aspect ratio of ~2.0.

Gall–Peters = Gall orthographic = Peters		Cylindrical	Equal-area	James Gall (Arno Peters)	1855	Horizontally compressed version of the Lambert equal-area. Standard parallels at 45°N/S. Aspect ratio of ~1.6. Similar is Balthasart projection with standard parallels at 50°N/S.
Central cylindrical		Cylindrical	Perspective	(unknown)	c. 1850	Practically unused in cartography because of severe polar distortion, but popular in panoramic photography, especially for architectural scenes.
Sinusoidal = Sanson–Flamsteed = Mercator equal-area		Pseudocylindrical	Equal-area, equidistant	(Several; first is unknown)	c. 1600	Meridians are sinusoids; parallels are equally spaced. Aspect ratio of 2:1. Distances along parallels are conserved.
Mollweide = elliptical = Babinet = homolographic		Pseudocylindrical	Equal-area	Karl Brandan Mollweide	1805	Meridians are ellipses.
Eckert II		Pseudocylindrical	Equal-area	Max Eckert-Greifendorff	1906	
Eckert IV		Pseudocylindrical	Equal-area	Max Eckert-Greifendorff	1906	Parallels are unequal in spacing and scale; outer meridians are semicircles; other meridians are semiellipses.
Eckert VI		Pseudocylindrical	Equal-area	Max Eckert-Greifendorff	1906	Parallels are unequal in spacing and scale; meridians are half-period sinusoids.
Ortelius oval		Pseudocylindrical	Compromise	Battista Agnese	1540	Meridians are circular.[2]
Goode homolosine		Pseudocylindrical	Equal-area	John Paul Goode	1923	Hybrid of Sinusoidal and Mollweide projections. Usually used in interrupted form.
Kavrayskiy VII		Pseudocylindrical	Compromise	Vladimir V. Kavrayskiy	1939	Evenly spaced parallels. Equivalent to Wagner VI horizontally compressed by a factor of $\sqrt{3}/2$.

Cartography

En.wikipedia.org. 2020. List Of Map Projections. [online] Available at: <https://en.wikipedia.org/wiki/List_of_map_projections> [Accessed 14 May 2020].

En.wikipedia.org. 2020. List Of Map Projections. [online] Available at: <https://en.wikipedia.org/wiki/List_of_map_projections> [Accessed 14 May 2020].

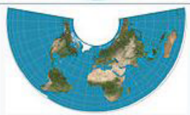
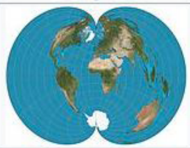
Robinson		Pseudocylindrical	Compromise	Arthur H. Robinson	1963	Computed by interpolation of tabulated values. Used by Rand McNally since inception and used by NGS in 1988–1998.
Equal Earth		Pseudocylindrical	Equal-area	Bojan Šavrič, Tom Patterson, Bernhard Jenny	2018	Inspired by the Robinson projection, but retains the relative size of areas.
Natural Earth		Pseudocylindrical	Compromise	Tom Patterson	2011	Computed by interpolation of tabulated values.
Tobler hyperelliptical		Pseudocylindrical	Equal-area	Waldo R. Tobler	1973	A family of map projections that includes as special cases Mollweide projection, Collignon projection, and the various cylindrical equal-area projections.
Wagner VI		Pseudocylindrical	Compromise	K. H. Wagner	1932	Equivalent to Kavrayskiy VII vertically compressed by a factor of $\sqrt{3}/2$.
Collignon		Pseudocylindrical	Equal-area	Édouard Collignon	c. 1865	Depending on configuration, the projection also may map the sphere to a single diamond or a pair of squares.
HEALPix		Pseudocylindrical	Equal-area	Krzysztof M. Górski	1997	Hybrid of Collignon + Lambert cylindrical equal-area.
Boggs eumorphic		Pseudocylindrical	Equal-area	Samuel Whittemore Boggs	1929	The equal-area projection that results from average of sinusoidal and Mollweide *y*-coordinates and thereby constraining the *x* coordinate.
Craster parabolic =Putniņš P4		Pseudocylindrical	Equal-area	John Craster	1929	Meridians are parabolas. Standard parallels at 36°46′N/S; parallels are unequal in spacing and scale; 2:1 aspect.
McBryde–Thomas flat-pole quartic = McBryde–Thomas #4		Pseudocylindrical	Equal-area	Felix W. McBryde, Paul Thomas	1949	Standard parallels at 33°45′N/S; parallels are unequal in spacing and scale; meridians are fourth-order curves. Distortion-free only where the standard parallels intersect the central meridian.
Quartic authalic		Pseudocylindrical	Equal-area	Karl Siemon Oscar Adams	1937 1944	Parallels are unequal in spacing and scale. No distortion along the equator. Meridians are fourth-order curves.
The Times		Pseudocylindrical	Compromise	John Muir	1965	Standard parallels 45°N/S. Parallels based on Gall stereographic, but with curved meridians. Developed for Bartholomew Ltd., The Times Atlas.

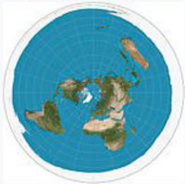
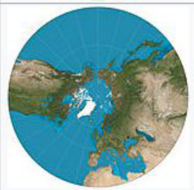
Cartography

En.wikipedia.org. 2020. List Of Map Projections. [online] Available at: <https://en.wikipedia.org/wiki/List_of_map_projections> [Accessed 14 May 2020].
En.wikipedia.org. 2020. List Of Map Projections. [online] Available at: <https://en.wikipedia.org/wiki/List_of_map_projections> [Accessed 14 May 2020].

Loximuthal		Pseudocylindrical	Compromise	Karl Siemon Waldo R. Tobler	1935 1966	From the designated centre, lines of constant bearing (rhumb lines/loxodromes) are straight and have the correct length. Generally asymmetric about the equator.
Aitoff		Pseudoazimuthal	Compromise	David A. Aitoff	1889	Stretching of modified equatorial azimuthal equidistant map. Boundary is 2:1 ellipse. Largely superseded by Hammer.
Hammer = Hammer–Aitoff variations: Briesemeister; Nordic		Pseudoazimuthal	Equal-area	Ernst Hammer	1892	Modified from azimuthal equal-area equatorial map. Boundary is 2:1 ellipse. Variants are oblique versions, centred on 45°N.
Strebe 1995		Pseudoazimuthal	Equal-area	Daniel "daan" Strebe	1994	Formulated by using other equal-area map projections as transformations.
Winkel tripel		Pseudoazimuthal	Compromise	Oswald Winkel	1921	Arithmetic mean of the equirectangular projection and the Aitoff projection. Standard world projection for the NGS since 1998.
Van der Grinten		Other	Compromise	Alphons J. van der Grinten	1904	Boundary is a circle. All parallels and meridians are circular arcs. Usually clipped near 80°N/S. Standard world projection of the NGS in 1922–1988.
Equidistant conic = simple conic		Conic	Equidistant	Based on Ptolemy's 1st Projection	c. 100	Distances along meridians are conserved, as is distance along one or two standard parallels.[3]
Lambert conformal conic		Conic	Conformal	Johann Heinrich Lambert	1772	Used in aviation charts.

Cartography

Albers conic		Conic	Equal-area	Heinrich C. Albers	1805	Two standard parallels with low distortion between them.
Werner		Pseudoconical	Equal-area, equidistant	Johannes Stabius	c. 1500	Parallels are equally spaced concentric circular arcs. Distances from the North Pole are correct as are the curved distances along parallels and distances along central meridian.
Bonne		Pseudoconical, cordiform	Equal-area	Bernardus Sylvanus	1511	Parallels are equally spaced concentric circular arcs and standard lines. Appearance depends on reference parallel. General case of both Werner and sinusoidal.
Bottomley		Pseudoconical	Equal-area	Henry Bottomley	2003	Alternative to the Bonne projection with simpler overall shape Parallels are elliptical arcs Appearance depends on reference parallel.
American polyconic		Pseudoconical	Compromise	Ferdinand Rudolph Hassler	c. 1820	Distances along the parallels are preserved as are distances along the central meridian.
Rectangular polyconic		Pseudoconical	Compromise	U.S. Coast Survey	c. 1853	Latitude along which scale is correct can be chosen. Parallels meet meridians at right angles.
Latitudinally equal-differential polyconic		Pseudoconical	Compromise	China State Bureau of Surveying and Mapping	1963	Polyconic: parallels are non-concentric arcs of circles.
Nicolosi globular		Pseudoconical[4]	Compromise	Abū Rayḥān al-Bīrūnī; reinvented by Giovanni Battista Nicolosi, 1660.[1]:14	c. 1000	

Azimuthal equidistant =Postel =zenithal equidistant		Azimuthal	Equidistant	Abū Rayḥān al-Bīrūnī	c. 1000	Distances from center are conserved. Used as the emblem of the United Nations, extending to 60° S.
Gnomonic		Azimuthal	Gnomonic	Thales (possibly)	c. 580 BC	All great circles map to straight lines. Extreme distortion far from the center. Shows less than one hemisphere.
Lambert azimuthal equal-area		Azimuthal	Equal-area	Johann Heinrich Lambert	1772	The straight-line distance between the central point on the map to any other point is the same as the straight-line 3D distance through the globe between the two points.
Stereographic		Azimuthal	Conformal	Hipparchos*	c. 200 BC	Map is infinite in extent with outer hemisphere inflating severely, so it is often used as two hemispheres. Maps all small circles to circles, which is useful for planetary mapping to preserve the shapes of craters.
Orthographic		Azimuthal	Perspective	Hipparchos*	c. 200 BC	View from an infinite distance.
Vertical perspective		Azimuthal	Perspective	Matthias Seutter*	1740	View from a finite distance. Can only display less than a hemisphere.

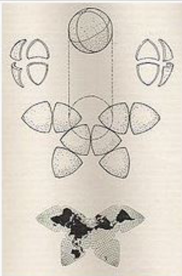
Cartography

Two-point equidistant		Azimuthal	Equidistant	Hans Maurer	1919	Two "control points" can be almost arbitrarily chosen. The two straight-line distances from any point on the map to the two control points are correct.
Peirce quincuncial		Other	Conformal	Charles Sanders Peirce	1879	Tessellates. Can be tiled continuously on a plane, with edge-crossings matching except for four singular points per tile.
Guyou hemisphere-in-a-square projection		Other	Conformal	Émile Guyou	1887	Tessellates.
Adams hemisphere-in-a-square projection		Other	Conformal	Oscar Sherman Adams	1925	
Lee conformal world on a tetrahedron		Polyhedral	Conformal	L. P. Lee	1965	Projects the globe onto a regular tetrahedron. Tessellates.
AuthaGraph projection	Link to file	Polyhedral	Compromise	Hajime Narukawa	1999	Approximately equal-area. Tessellates.
Octant projection		Polyhedral	Compromise	Leonardo da Vinci	1514	Projects the globe onto eight octants (Reuleaux triangles) with no meridians and no parallels.

Cartography

En.wikipedia.org. 2020. List Of Map Projections. [online] Available at: <https://en.wikipedia.org/wiki/List_of_map_projections> [Accessed 14 May 2020].

En.wikipedia.org. 2020. List Of Map Projections. [online] Available at: <https://en.wikipedia.org/wiki/List_of_map_projections> [Accessed 14 May 2020].

Cahill's butterfly map		Polyhedral	Compromise	Bernard Joseph Stanislaus Cahill	1909	Projects the globe onto an octahedron with symmetrical components and contiguous landmasses that may be displayed in various arrangements.
Cahill–Keyes projection		Polyhedral	Compromise	Gene Keyes	1975	Projects the globe onto a truncated octahedron with symmetrical components and contiguous land masses that may be displayed in various arrangements.
Waterman butterfly projection		Polyhedral	Compromise	Steve Waterman	1996	Projects the globe onto a truncated octahedron with symmetrical components and contiguous land masses that may be displayed in various arrangements.
Quadrilateralized spherical cube		Polyhedral	Equal-area	F. Kenneth Chan, E. M. O'Neill	1973	
Dymaxion map		Polyhedral	Compromise	Buckminster Fuller	1943	Also known as a Fuller Projection.
Myriahedral projections		Polyhedral	Compromise	Jarke J. van Wijk	2008	Projects the globe onto a myriahedron: a polyhedron with a very large number of faces.[5][6]
Craig retroazimuthal = Mecca		Retroazimuthal	Compromise	James Ireland Craig	1909	
Hammer retroazimuthal, front hemisphere		Retroazimuthal		Ernst Hammer	1910	

Cartography

Hammer retroazimuthal, back hemisphere		Retroazimuthal		Ernst Hammer	1910	
Littrow		Retroazimuthal	Conformal	Joseph Johann Littrow	1833	on equatorial aspect it shows a hemisphere except for poles.
Armadillo		Other	Compromise	Erwin Raisz	1943	
GS50		Other	Conformal	John P. Snyder	1982	Designed specifically to minimize distortion when used to display all 50 U.S. states.
Wagner VII = Hammer-Wagner		Pseudoazimuthal	Equal-area	K. H. Wagner	1941	
Atlantis = Transverse Mollweide		Pseudocylindrical	Equal-area	John Bartholomew	1948	Oblique version of Mollweide
Bertin = Bertin-Rivière = Bertin 1953		Other	Compromise	Jacques Bertin	1953	Projection in which the compromise is no longer homogeneous but instead is modified for a larger deformation of the oceans, to achieve lesser deformation of the continents. Commonly used for French geopolitical maps.[7]

Perspectographs

An instrument for obtaining, and transferring to a picture, the points and outlines of objects, so as to represent them in their proper geometrical relations as viewed from a single point.

Albrecht Dürer's grid

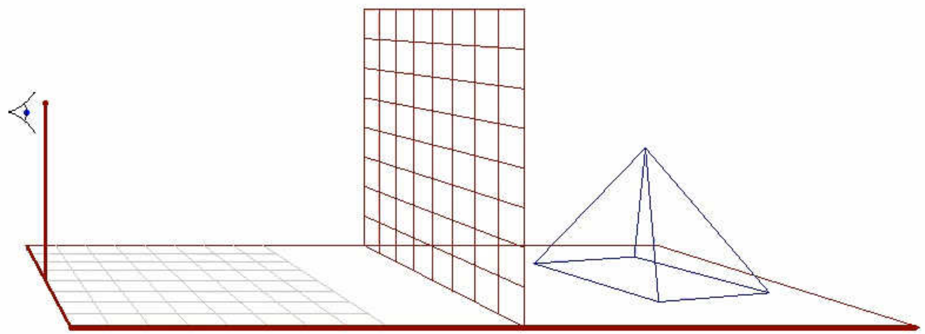
Here is how Dürer explains the functioning of this device:

Here is another method to do portraits. This allows representing any body, whatever the wanted size is, bigger or smaller than the real one. It is better than the glass because it allows more freedom. We need a frame with a grid made of black and solid strings: each square measures 2 cm on the side. Then we need an ocular, whose height is changeable. This is the eye O. Place the object you want to draw far enough and in a position you like. Go back and look through O and check if the position is fine. Then, place the grid between the object and the ocular as described below. If you want to use just a few squares of the grid, place the object as close to the grid as possible. Then draw another grid, either big or small, on the surface (paper or canvas) where you want the image to be. Look at the object from the top of the ocular and draw on the grid what you see in each square. This is the correct procedure. The FIGURE below represents the device.

Dürer then continues:

If anyone wants to paint a giant on the sides of a tall tower starting from a small image, putting together a sufficient number of papers to obtain a big enough grid would be very inconvenient. Therefore do not build a big paper grid but cut many pieces of paper, each one of the dimensions of the squares this big grid would have. Draw them one after the other as indicated before, in the correct order; then you will put them together, as you do with cards, and, when working on the wall you can copy them one after the other in the correct order without having to draw the giant in only one go. The reproduction of drawings through a grid was common practice among cartographers and painters for a long time. Masaccio left a clear example of how orthogonal grids were used to enlarge drawings for frescos (we can see traces on theface of the Virgin in Trinità.(1) Grids projected from a punctiform source were used to obtain anamorphoses (2) or to produce illusionistic decorations of vaults and walls (3).

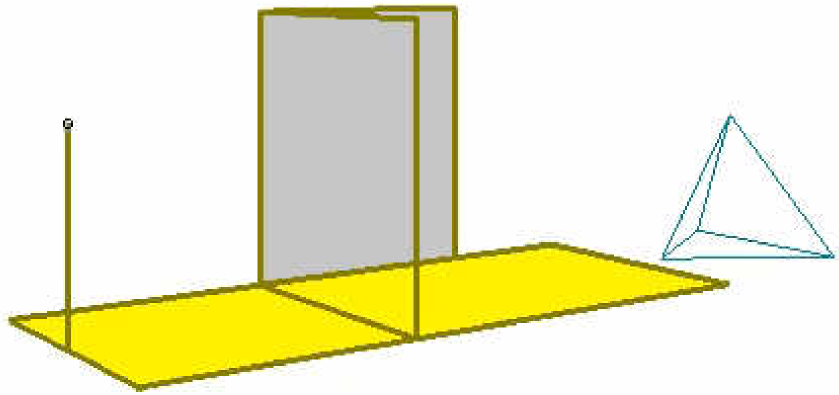
(1) F. Camerota, "Nel segno di Masaccio", cat. Giunti 2001, IX 1 (La "terza regola")
(2) Cf. e.g. S. Stevin, "De Sciagraphia", Leyden 1605 (ed. R. Sinisgalli, Il contributo di S. Stevin allo sviluppo della prospettiva artificiale, Roma 1978, pag. 332 - 335); J.F. Niceron, "Thaumaturgus opticus…", Parigi, 1646
(3) A. Pozzo, "Perspectiva Pictorum et Architectorum", Roma, 1693 - 1700.



Archiviomacmat.unimore.it. 2020. Home. [online]
Available at: <http://archiviomacmat.unimore.it/PAWeb/Sito/Inglese/templatei.htm> [Accessed 1 May 2020].

Albrecht Dürer's Perspectograph Door

It is very likely that this perspectograph was invented by Dürer (as opposed to the other three devices he describes). The use of a string (linking the eye - substituted by a nail - to the contour of the object) to represent the light ray is of particular interest. This device is a mechanical model of both the visual pyramid and the propagation of light. When the door is open the frame is virtual: it is represented by the points of intersection between two mobile strings. When this intersection belongs to the light ray (i.e. to the string representing it), it is the image of the point of the object from which the light ray comes from (or that is hit by the light ray) and will then be marked on the real frame (i.e. the closed door). You do not need numerical coordinates. Two people are needed in order to do a perspective drawing with this device. While an assistant moves the (visual) light ray along the contour of the object, the artist identifies the intersections with the virtual frame and marks them on the real frame. The human eye is eliminated; vision is regulated via geometric and mechanical rules; people become more or less passive instruments. Dürer's instructions about how to construct and use the perspectograph, are very detailed. Here is the last part: Put the lute (…) at the set distance from the frame and be careful: it must remain still for all the time you need. Ask your assistant to keep the string hanging from the nail straight (at the other end there is a weight) and to touch all the main points of the lute. When he stops at one of those points (the string being straight) you must move the other two strings (the ones fixed to the border of the frame) so that they intersect with his string. (In order to fix them in this position) paste to the frame also the endpoints of the two other strings, tell your assistant to loosen his string. Now close the door and mark on the frame the intersection point of the two remaining strings.



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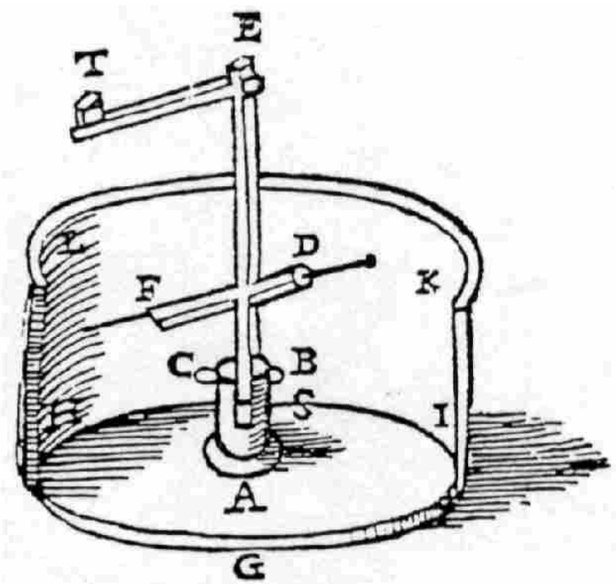
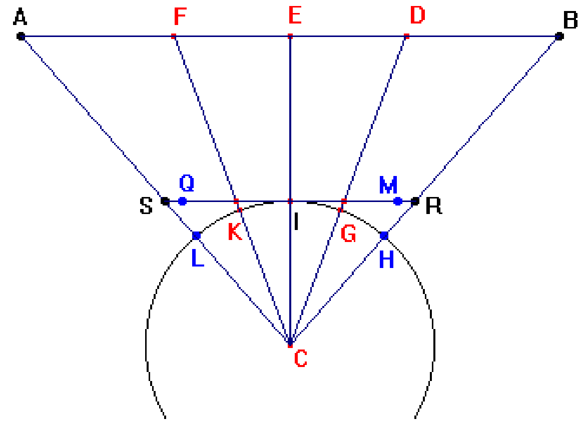
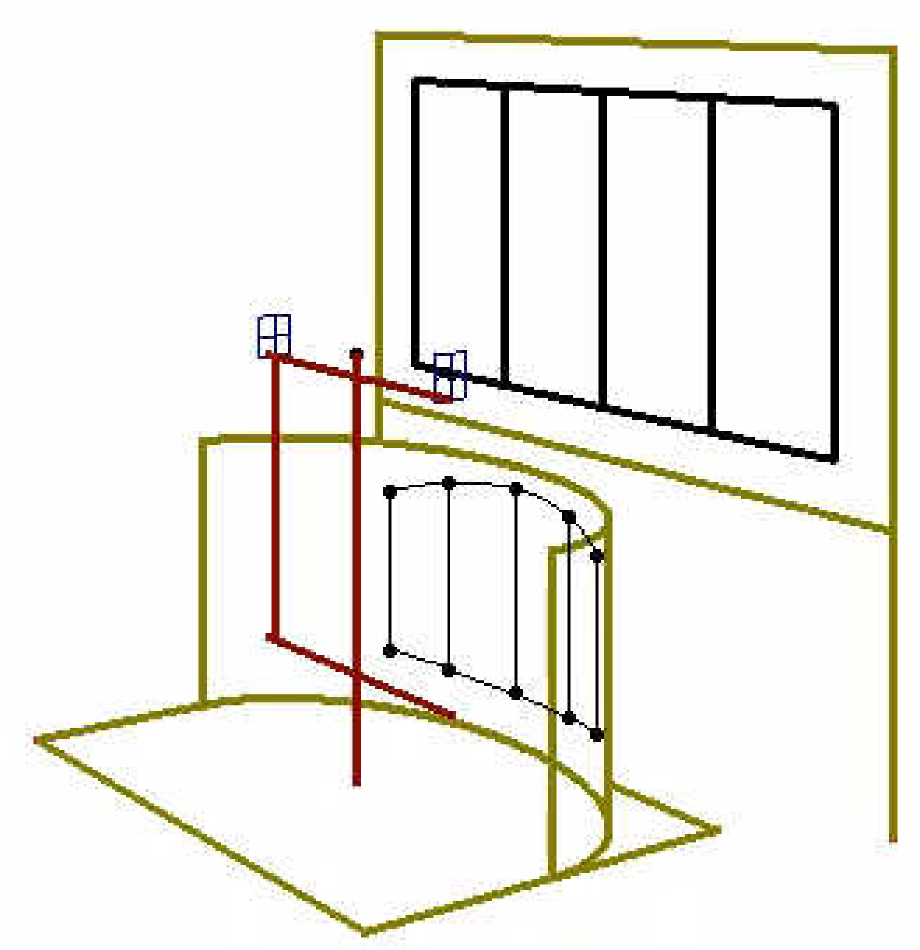
B.Lanci's Perspectograph

Baldassarre Lanci was mainly a practitioner. He was born in Urbino at the beginning of the 16th century and worked in the construction of many important fortifications in Lucca, Livorno, Nettuno, Paliano, Civitavecchia, Roma, Ancona. He built the fort in Siena (1561), in San Martino al Mugello (1569) and in Radicofani. He also planned the church of S. Maria della Rosa in Chianciano (1565). He died in Firenze in 1571. His perspectograph uses a "circular surface equidistant from the eye" rather than "a plane intersecting the visual cone": more precisely, he uses a cylindrical surface tangent to a sphere with centre in the eye (represented by a pivot). We can find a model of this perspectograph in the Museum of Science in Firenze (catalogo Electa, MI 1968, fig.90); here we find a reconstruction of the model according to the instructions from C. Maltese in "La prospettiva Rinascimentale", a cura di M. Dalai Emiliani, FI 1980, p. 417

This device is mentioned in "Pratica della Perspectiva" by D. Barbaro (Venezia, 1569) and in "Due regole della Prospettiva Pratica " by J. Barozzi (commentaries from E. Danti, Roma 1583). This is the description presented by Danti, referring to a FIGURE he sketched (see figure 1 on the left): In the middle of the round table GHSI a pivoting foot around A sustains a rod SE, nailed in C and B that will be able to rotate together with the foot and the nail. At the top of the rod, a diottra ET allows sighting the objects to be drawn in perspective; further down, a small copper rod DF, pivoted on SE and crossed by a rod, is kept always parallel to ET.
Fig. 1 Danti criticises this device, observing that (when the paper where the object has been drawn is laid out on a plane) it provides false images: because when the curved surface LKIGH on which segment AB is projected (see figure 2) is laid out flat, we will see intervals that appear smaller towards the extremities. The intervals increase towards the extremities as AF or DB will appear equal to FE and ED. On the other hand, in the traditional way, when using a flat plane SR equal intervals will appear equal and different intervals will appear different (Maltese, op. cit.). However, Danti does not realise that a very important category of ratio was represented on Lanci's cylindrical surface, that is horizontal angular quantities (Maltese, op.cit.).The misunderstanding is probably due to the fact that Lanci had mainly practical objectives (triangulations), while Danti was more interested in scenic needs that required nice visual effects rather than practical use of images.







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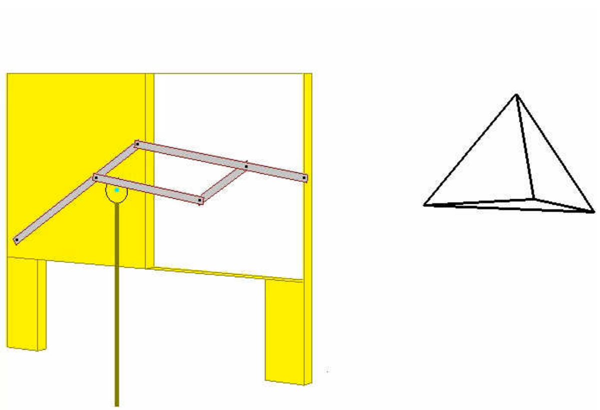
C.Scheiner's Perspectograph

The treatise "Pantographice seu ars delineandi" written by padre Scheiner (Jesuit), German astronomer (know also for his relationship with Galilei), had many Italian versions (see for example the one by G. Troili, Bologna 1653). In the introduction Scheiner tells us how he came to discover his perspectograph. In 1603 in Dillingen an excellent painter (it is Pierre Gregoire – Petrus Gregorius – author of "Syntaxeon artis mirabilis, Leyda 1575) shared with Scheiner his proud of having invented a drawing machine, but refused to disseminate the secret. Gregorius said that he did not believe that such a thing could even be imagined; in fact, that was not human invention but divine inspiration and he believed that it had been revealed to him by some celestial genius. He only revealed to the astronomer that his instrument was based on the use of compasses with a fixed center …. Scheiner started working on his own and after many attempts he finally produced a very clever and flexible device aimed at copying, enlarging and reducing drawings, representing objects in perspective and doing anamorphoses. (Cf. M. Kemp, "La scienza dell'arte", Giunti, 1994) The key idea at the basis of Scheiner's perspectograph (which is the use of an articulated parallelogram to enlarge or reduce bi-dimensional images and keep proportions) is still at the basis of the construction of modern pantographs, which are more sophisticated only from a mechanical point of view. In the perspectograph, the parallelogram is mounted vertically on a wooden board, supported by a pivot AC fixed in C and has an index MC (the index is a pointer that moves along the contour of the object to be represented) and a pen BP that traces the drawing. During the deformation of the parallelogram (that is moved by the operator through the pen BP), points C, I and P remain collinear (the distances CP and CI determine the scale factor).However, because the image touched by the index is not real, but is only the representation of the object on the surface of the visual cone, and the copy that the pen draws must be real, plane KHNQ must be partly real, as is LQNO, and partly rational and mathematical, as in KLHO. It is real in the part that is touched by the pen, rational in the part that is touched by the index. Finally, we need to decide where to place the eye while we operate the device; the eye must stay fixed in the same point. Then we can operate the device. This perspectograph, that (as Dürer's door) inspired many analogous devices, as for example Wren's (1669) and Watt's (1765), has a particular historical importance: - The construction of perspective images is completely mechanical - as it is with Cigoli's perspectograph.- The hole through which we look (moving this, the eye can see the apparent contour of the object) describes, within the "empty" window, a virtual image (intersection of the visual pyramid with the plane of the window) that the pantograph translates (on the paper attached to the frame) in a real drawing. This automatic passage from virtual to real is considered by all commentators at the time an "admirable", nearly magic, characteristic of this device. - The relative dimension of the image can be changed without having to change the distance between the viewer's eye and the intersection.

- Nowadays we know that the pantograph performs homotheties. Königs (Lezion di cinematica", 1897) writes: Each epoch has got - without being aware - the inventions of future epochs; the history of things often anticipates the history of ideas. When Scheiner published the description of his pantograph in 1631 for the first time, he certainly did not know the general concepts it contained. He could not know them because they are related to the theory of transformations which was developed only in the 20th century.



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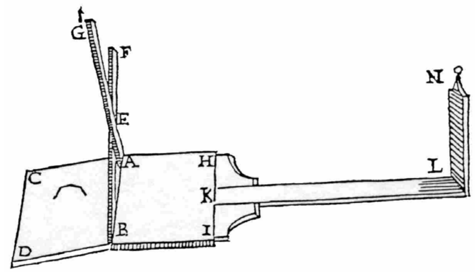
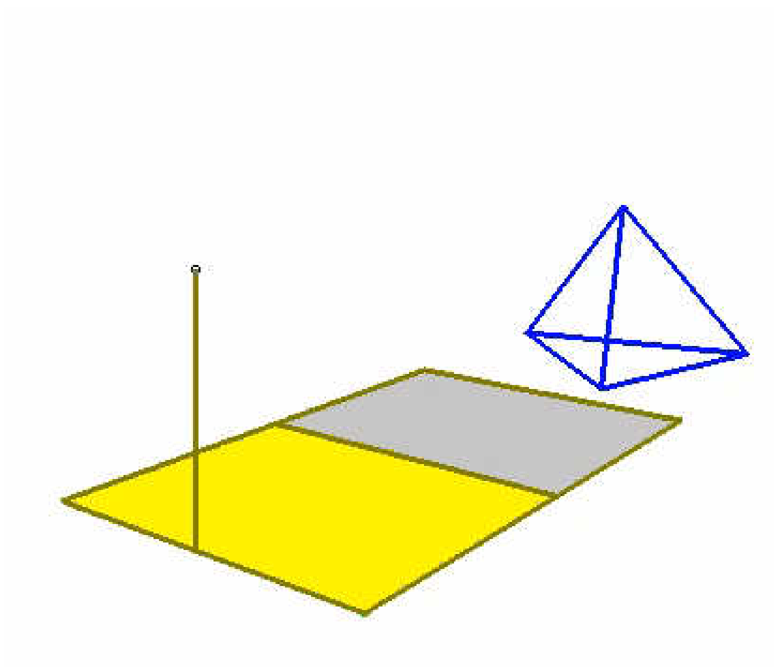
D.Girolamo da Perugia's Perspectograph

Let E. Danti talk:
Through practice I came to understand how useful Albert's door is …. So I wanted to represent this device shown to me in part by Reverendo Don Girolamo da Perugia, Abate di Lerino, because it seems easier to use than the other two I talked about previously….



The figure is very schematic. More details about how the device is constructed are presented in the description below. Prepare two wooden boards of the same size BC and BH, flat; link them together in A and B so that BH stays still and BC can be moved up to forming a right angle with BH. In A and B fix two wooden or brass rules in a way that they can move and intersect, substituting Albert's strings; then use another rule KL that can be moved towards A and B or in the opposite direction according to where you want the distance point to be with respect to the two rules, that represent the frame. In L fix a rule LN of the same size as BD. This is the frame. In L fix a rule LN of the same size as BD. This is the last step. This device works in the same way as the other two, with the exception that with your eye in N, you will sight the object you want to draw by moving the two rules AG and BF until the visual ray coming from the eye goes though their intersection point E (you will copy this point on the door, after bringing it up) …. This perspectograph is more exact than Dürer's door. Drawing is very difficult with the door when you have strings: because when the radial string touches the transversal ones, it can move them from the correct position and lead to quite big errors. But when instead of strings you have rules the error is much smaller, therefore I have always considered this device as the best one …. The extracts above are taken from p. 57 and 58 of "Commentari alle due regole della Prospettiva del Vignola…", Regola I, Cap. III, annotazione prima. (ed. Zannetti, 1583).



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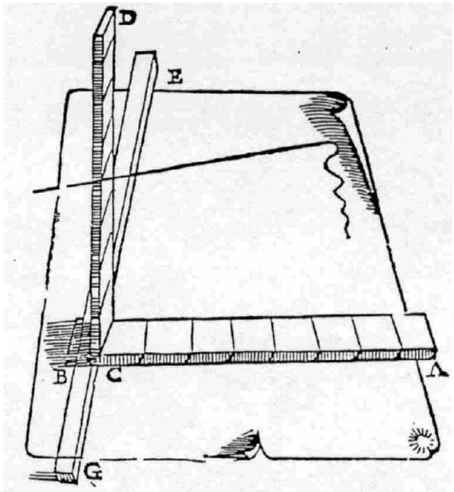
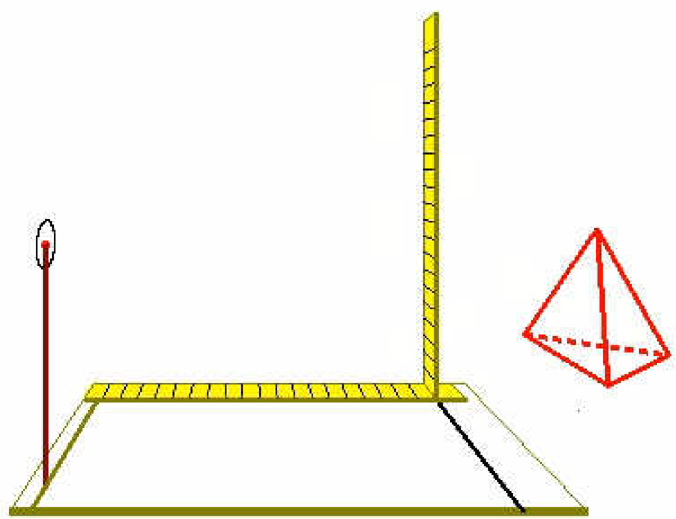
E. Danti's Perspectograph (with graduated rods)`

We add this device to the ones previously mentioned. It is made of rules and I constructed it some time ago in Florence, as described below. I prepared three rules about 1 meter long: AC and CD are the same size and divided into the same number of parts (I decided to divide them into 40 parts). I connected them in C so that they form a right angle, AC being the same length as CD; AC continues in CB, which forms a right angle with EG; AC and CD are free to move under EG, which represents the width of the door whereas CD is the height.



Once the device is constructed, it can be operated in the same way as the others. With the string (…) you will touch the object you want to draw in Perspective, moving the rod CD (together with CA) back and forth from E to G until a side of CD touches the string or the visual ray; on this side you will observe which point of the graduation the string is close to and then you will find the same point on AC, and next to this you will draw a point on the paper, which is attached to the board under the device. On this paper you will trace everything that would have been previously traced on the door: you will see how much more comfortable drawing on a paper fixed to the board and using mobile rules is. We need to remember that EG must be still on the table so that CD, which substitutes the frame intersecting the visual pyramid, does not change its position (with respect to the eye) and represents what Nature allows us to see. But with this door, as in the one we will describe later on (Vignola's) you will need a bit of practice in the case when the string does not hit exactly one of the graduation lines of string does not hit exactly one of the graduation lines of CD: this case however can happen also with the device described before (Trigini de' Marij's). Therefore the best device will certainly be the one constructed by Abate di Lerino.



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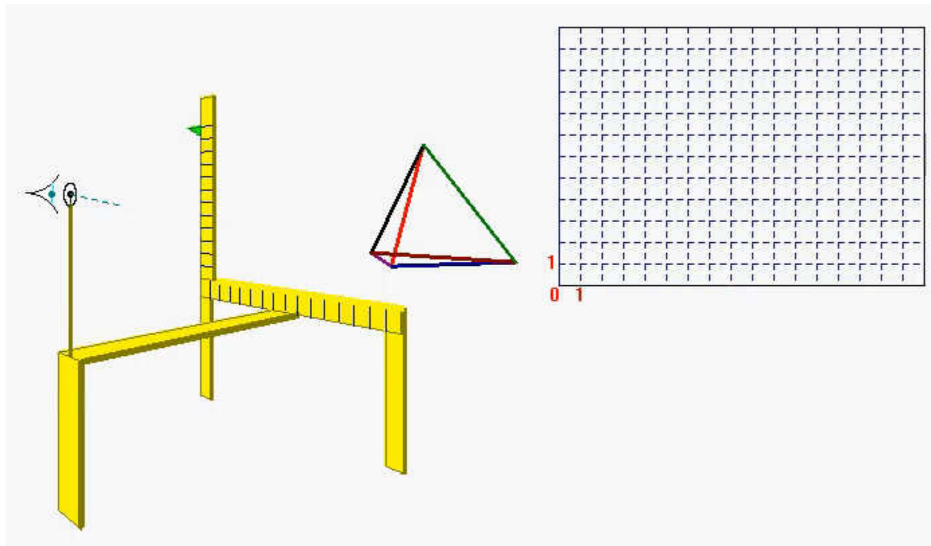
J.Barozzi's Perspectograph

Here is the description by Danti. I want to describe this device, that I saw represented in a sketch (without any description) by Vignola, so that we can see the variety of instruments derived from the door and representing the plane that cut the visual pyramid. In this case, the base AB and the rule CD represent the door: they have the same role played by rules EG and CD in the previous device (Danti's perspectograph). Even though this figure is very clear and does not require any comment, I want to make clear that MN must

be free to move up and down according to where you want the eye to be. However, once the wanted height of the eye is found, N must stay still until we are finished with the drawing. The slide AB can be placed more or less close to MN according to which distance we want the eye from the frame. In the same way, MZ can be pushed left or right, according to the side we want to look at the object from. Once decided these positions, you look at the object form the sight, rotating L with your hand so that CD, which comes from HFG, moves towards A or B, until the ray connecting the object to the eye intersects CD. Mark the point where CD is intersected by the ray and point C where CD intersects BA. (We would say: determine the Cartesian coordinates of the intersection point in a Cartesian reference system centred in A or B). Then attach a sheet of paper to the board, with a grid with as many squares as the divisions of CD and BA are; on the two sides mark the numbers that you can see on AB and CD. While you operate the device, mark on the paper the points of intersection. When the string does not touch exactly one of the divisions, some practice is required in order to determine the corresponding point on the paper. This does not happen neither with Albert's door nor with the 'diottra' nor with Abate di Lerino's perspectograph: these devices would have been sufficient. But in this book I wanted to describe the other devices too, so that it comes out how perspectograph: these devices would have been sufficient. But in this book I wanted to describe the other devices too, so that it comes out how those three really excel.v



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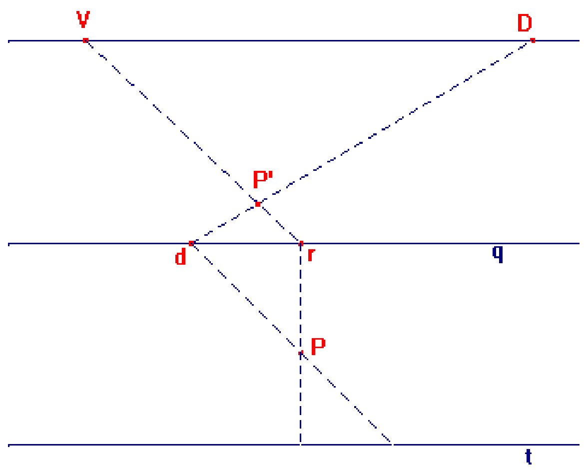
J. Barozzi's Bi-Dimensional Perspectograph

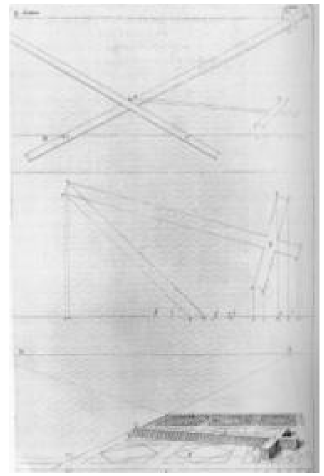
A. "Le due regole della prospettiva pratica" by Jacopo Barozzi da Vignola, written between 1527 and 1545 and printed after his death, with the "Commentari" by Egnatio Danti, in 1583, is an important volume for a number of reasons: the extremely large diffusion it had (eight editions by 1700 and many others afterwards); the fact it was written by two people, a painter-architect expert of mathematics (Barozzi) and a mathematician-cartographer expert of painting (Danti); because it contains a very clear exposition of the rules for using distance points that had never been expressed as clearly despite having previously been used by practitioners and theorists. (1) The collaboration between the two (even though it happened "at a distance" as Barozzi was dead when Danti started his commentaries) was particularly fruitful because of the singular equilibrium reached between the specialist knowledge of one complemented by the knowledge of the other. Barozzi decided to write this volume when, while busy with preparing designs for the construction of wooden tarsia, he had to face the difficulties of constructing perspective images without having any reference book to consult in order to solve the spatial-geometrical problems he had (2): he did not know about the existence of the treatises available at the time, of not easy access and reading. On the other hand, Danti was professor of Mathematical Science first in Bologna and then in Florence and he knew the treatises about perspective. Therefore the volume produced by these two people is very clear and rigorous and the practical and theoretical needs are nicely mediated.
B. In Ch.XI of the part of the treatise dedicated to the Second Rule, Barozzi describes a device to draw in Perspective that can make the artists' job simpler. The beginning of the chapter (How to draw in Perspective with two rulers, without having to trace many lines) reveals the utility of his invention. If all lines required by the Second Rule were traced (they are dead lines, i.e. they need to be erased after the construction is finished), there would be too much confusion. By using two rulers, one fixed in the vanishing point and the other one in the distance point (both on the horizon line) all the dead lines can be avoided.
C. In order to understand the functioning of this device, that works in two dimensions and therefore is included in the two-dimensional perspectographs, we first need to know the Second Rule that (as Barozzi writes) works in a similar way to the First one but is easier to use.

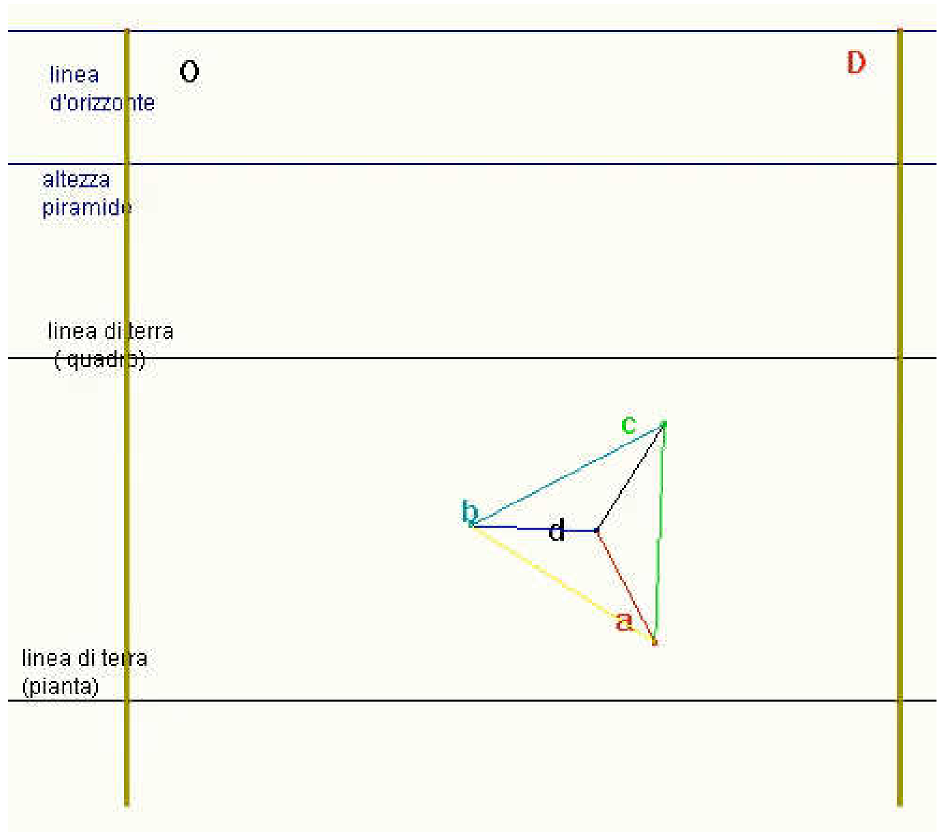
Nomenclature:
The diagonal line is a line parallel to the ground plane (floor) and forming a 45° angle with the frame: diagonal lines converge to the distance point in the perspective image;The straight line is a line parallel to the ground plane (floor) and perpendicular to the frame (wall): straight lines converge to the vanishing point in the perspective image (viewpoint) The diagonal points are points in which the diagonal lines on the floor meet the ground line. The straight points are the points in which the straight lines on the floor meet
the ground line. In order to draw the perspective image of any point P on the floor you must follow the following procedure (cf. Figure):

1. trace on the sheet of paper two parallellines t, q , both representing the groundline: the first one (the one at the bottom)is the origin of the "ground semi-plane"that is behind the frame; the second one (the one at the top) is the base of theframe;
2. a third line (above the first two and parallel to them) represents the horizonline; on this one we choose the point of view (e.g. V) and the distance point(e.g. D)
3. place point P (object to draw) in a given position behind the frame
4. trace from P a diagonal line (in the opposite direction to D) and a straight lineand determine their intersections with t, so obtaining the diagonal point d andthe straight point r
5. trace lines Dd and Vr: their intersection P' is the perspective image of P.We can observe that for each point we need four dead lines; but the operationsdescribed in 4 can be done by using a square and compass, tracing on q only the diagonal and straight points; moreover, the operations described in 5 can be donethrough two thin rulers fixed in D and V, without having to trace the dead linesDd e Vr.This is Barozzi's invention.

(1) L. Vagnetti, De Naturali et Artificiali Perspectiva, Firenze, 1979.
(2) L. Vagnetti, Op. Cit., pag. 321







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J.H.Lambert's Bi-Dimensional Perspectograph

A. Jean-Henri Lambert was born in Moulhouse in 1728 : he taught in Switzerland and was a member of the Accademia Prussiana di Scienze di Berlino, where he died in 1777. His interests are not limited to perspective. He was involved in numerous activities (mathematics, physics, astronomy, geography), and his books show the intention to present all his work as unitary and following general ideas. This is exemplified for example when he links his interests as cartographer to the problems of perspective: A map should maintain the same relationship with regions, hemispheres and the earth as engineering drawings have with building, garden, forests. But the terrestrial globe has a spherical surface …(1). Or, for example when he looks for the foundation of the possibility that a perspective device embodies geometric rules in the coherence of these rules with some general principles of continuity that dominated mechanicistic philosophy at the time: We know that it is technically possible to make a machine execute a continuous and regular movement: both in the case this movement is uniform and if it is repeated at regular intervals. Let use examine whether these conditions of regularity are realised during the prospective projection of a plane … (2). Other important characteristics of Lambert's work are its sensitivity towards the economical context and its attention towards practical activities. B. Lambert's first work dedicated to perspective(3) (written in 1752 and for a long time remained manuscript) contains the description of a mechanism aimed to draw in perspective the base of buildings (gardens, fortifications, …). Among the mechanisms operating in two dimensions, this is the first one that is "automatic". This perspectograph requires the person operating it only to move the punch on the drawing to be transformed, which is different from what one was required to do for example in the case of Barozzi-Danti's perspectograph. The pages dedicated to this mechanism are interesting because of the intersection between theory and practice, between science and technique: the construction of the perspectograph is at the same time a proof of a series of statements that justify it functioning.(4). Below we illustrate how the perspectograph works and the properties it embodies.

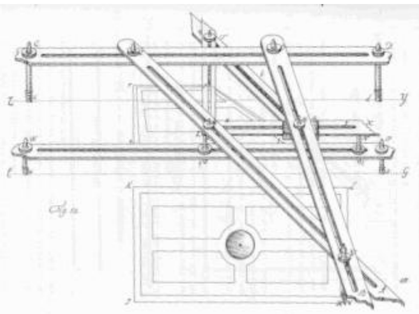
Let us consider Figure 1, where and are per-spective planes; O is the viewpoint, P* is the ima-ge of P, t is the*

ground li-ne, l is the limit line, S and L are (respectively) the orthogonal projections of O and P* onto . Points P, L, S are colline-ar. Let us project O* and P* in O and M from an improper centre with rays parallel to*

and forming a 45° angle with ; M and O are collinear with P. Let us project P in P' with a ray forming a 45° angle with and and contained in a plane perpendicular to t, such that P and P' are on opposite sides of t (so*

LMP' is a right-angled isosceles triangle). The configuration obtained on (Figure 2) is the one realised mechanically by Lambert's perspectograph. On the board there are two straight slides t and l; S and O are two pivots fixed on l; q, p, s and r are metallic rods with a straight slide places as follow:

rod s is hinged in S and obliged to go through L, cursor moving in t; rod q, hinged in O, is obliged to go through M, cursor moving in t; rod p is fixed to cursor L so that it stays perpendicular to t; rod r is fixed to cursor M so that it always forms a 45° angle with t; Given a point P, make q and s go through it: point P', intersection of p and r, is the perspective image of P. At any instant, LM = LP'; in Figure 1 LP' = LP. This makes sure that the figure constituted by the points P* (on the frame) is congruent to the one formed by the points P' on the board (ground plane). We can also notice that the figures formed by points P and P' are corresponding figures in a homology (a concept that was not yet known at the time of Lambert).*



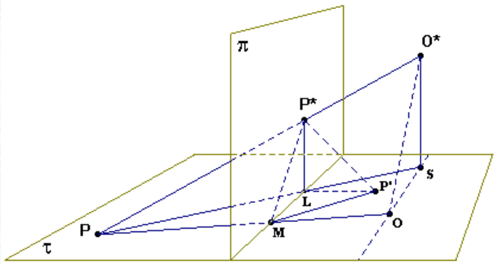


Fig. 1

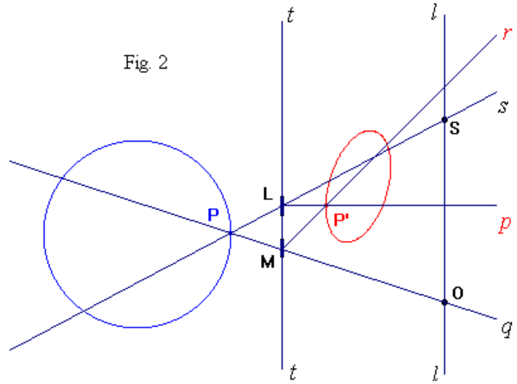
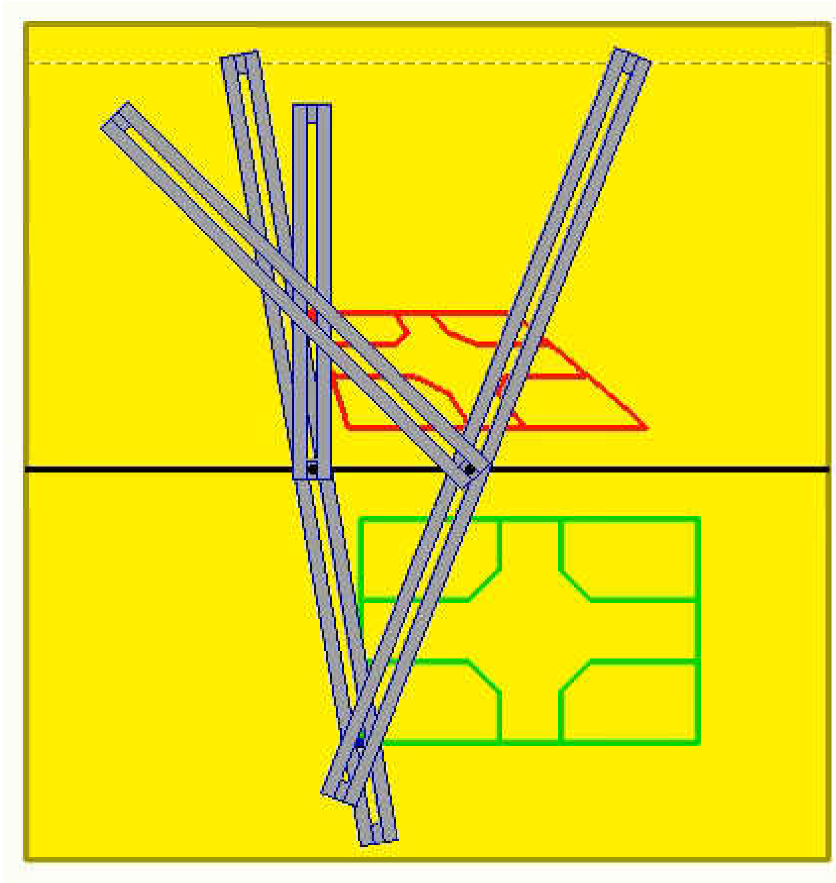


Fig. 2

(1) J. H. Lambert, Notes and Comments on the Composition of Terrestrial and Celestial Maps, trad. Tobler, Michigan 1972.

(2) J.H. Lambert, Essai sur la Perspective, trad. Peiffer, paragraph 21, p. 19.

(3) J.H. Lambert, Anlage der Perspektive, manuscript 1752, first published in M. Steck, J.H. Lambert Schriften zur Perspektive, Berlino 1943.



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J.H.Lambert's Bi-Dimensional Perspectograph 2

Lambert's main work concerned with perspective, Freye Perspektive, was published in Zurich in 1774, in an English and French edition at the same time. The aim of the author was to free perspective constructions from the need to first draw orthogonal projections of the object at stake; Brook Taylor had already tackled this problem in a treatise in 1715 (that Lambert did not know) (1). In the third part of Freye Perspektive, Lambert describes a number of devices to simplify perspective drawing. Among those we also find a "deforming pantograph", which is a simplification of the one described in Anlage zur Perspektive (Lambert's model 1), but with the same function, that is to automatically draw the base of buildings in an accurate perspective. In order to explain the functioning of this machine (and the properties it embodies), we start from a three-dimensional space and make a projection on the ground plane. Figure 1 represents a perspectivity between planes and . O is the viewpoint; P* is the image of P; t and l are, respectively, the ground line and the horizon line; S* is the foot of the perpendicular from O to . Let us project points O*, S* and P* respectively to O, M and P' from an improper centre, with parallel rays forming a 45° angle with and so that M and O belong to*
a line perpendicular to t and P' is on the opposite side of P with respect to t. Because P, P and O* are collinear, so are P, P' and O. The configuration that is obtained on (P and P' corresponding movable points, O*
and M fixed points) is the one that is mechanically realised in this version of Lambert's perspectograph (see Figure 2). On the base of the model there is a straight slide t, and two pivots O and M (such that OM is perpendicular to t). The metallic rods a, b and r are set as follows:

- *Rod a is hinged in O*
- *rod b is hinged in M and fixed*
- *to a cursor T moving on t*
- *rod r is hinged to cursor T so*
- *that it always stays perpendicular to t.*

Given a point P on the base of the model, let use make rods a and r go through this point; point P', intersection of b and a, is theperspective image of P. It is clear that the figures formed by points P and the figures formed by points P' are congruent. On the other hand, the correspondence between points P and P' is a homology (this concept was not known at the time of Lambert).*

(1) B. Taylor, Principles of Linear Perspective, or the Art of desining upon a plane the representation of all sortes of objects, as they appear to the eye, Londra 1715.

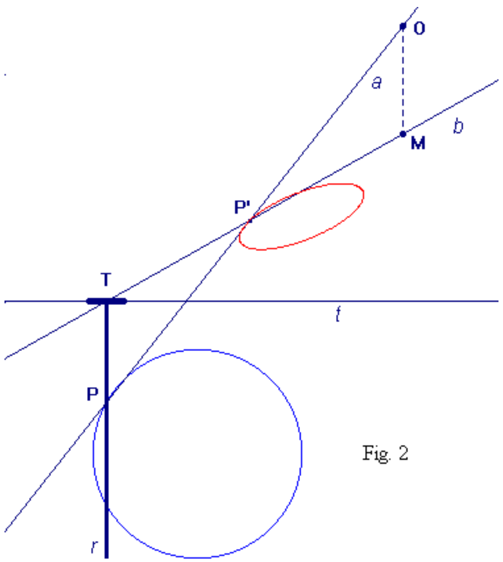
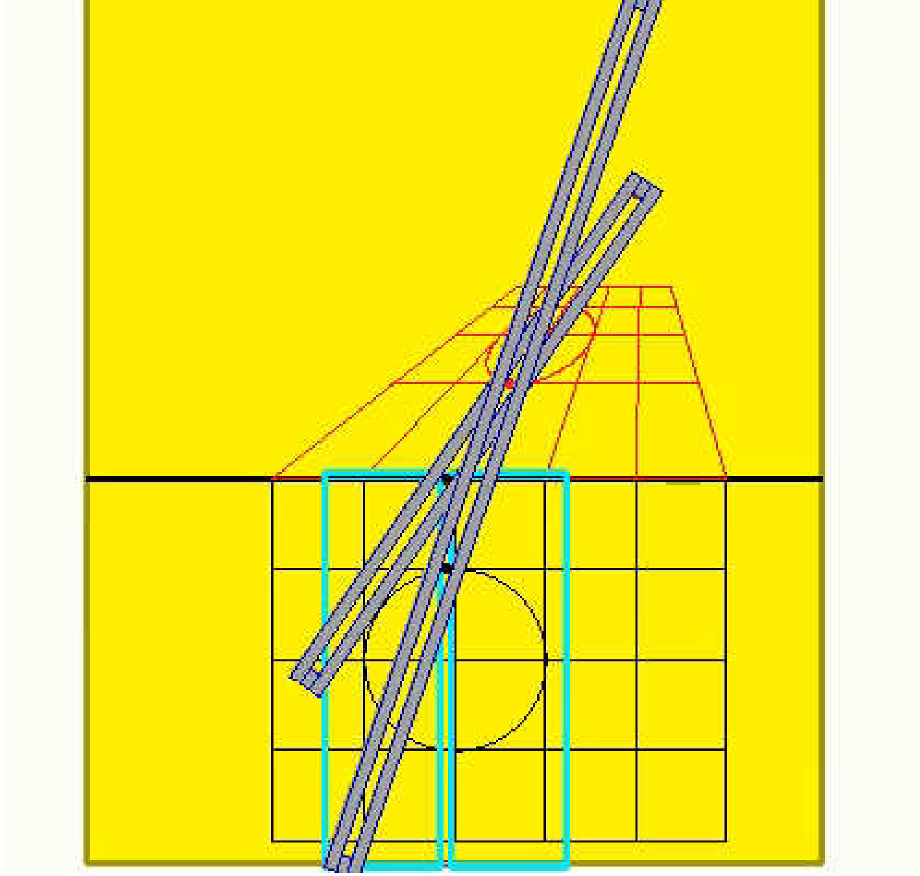


Fig. 2



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J.Keser’s Perspectograph

Dürer tells us that Jacobus Keser, after studying his perspectograph, invented a quicker and better method to draw the perspective image of an object. Therefore, Dürer goes on saying that because it is a rapid way, I feel obliged to teach it for the sake of everyone and to honor the intelligence of Mr Keser. We report the arguments used by Dürer to clarify the aims of Keser’s invention, because they reveal two interesting points: first of all, an awareness of the possible deformations that can be produced in perspective drawings if eye, frame and objects are not adequately placed with respect to one another; second, the constant preoccupation about making practical operations as easy as possible and shorten the working time, always following geometric rules.

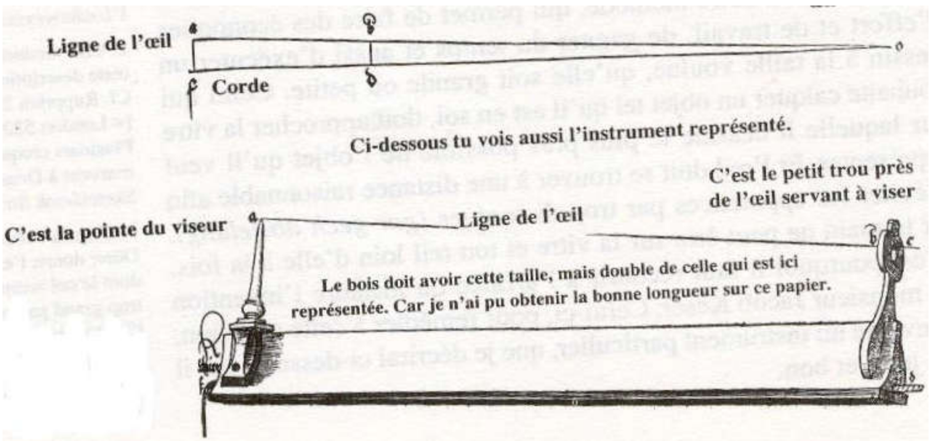
Each bi-dimensional object can be represented on a glass close to the eye. However, there are objects that appear deformed because of a particular perspective due to the fact that the eye is too close to the glass. In fact, the parts that are closer to the eye appear to be much bigger than the ones that are further away. When the object is far from me, the drawing on the glass will be very small and I cannot push the glass too far away from me because I need to be able to reach it with my hand. If on the other hand I put the glass close to the object and place my eye at a reasonable distance, I will not be able to reach the glass with my hand. That is why it is better to follow another method, that is quicker and easier and gives the possibility to represent objects in the size we want, being it big or small. Whoever wants to draw an object preserving proportions must put the glass, on which he wants to make the drawing, as close as possible to the object to be represented. And his eye should be at a correct distance so that there are no distortion effects. But it cannot be that your hand is on the glass and at the same time your eye is far from the glass. That is why we need to use the device created by Jacob Keser.

As in the door, the eye will be represented by a nail and a cord will be attached to it, thin and strong, made of silk, as long as needed. This cord does not represent the visual ray but only the support for a sight which is made of three parts:

- a wooden rule with a triangular section, with a hole in the middle which will accommodate the cord;
- a pointed viewer placed at one of the end points of the rule;
- an eye-piece at the opposite end point of the rule.

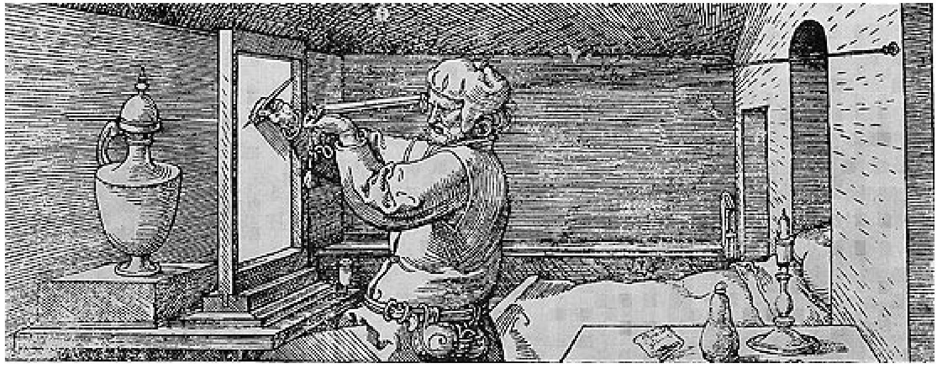
Dürer sets the dimensions of the rule and all the other parts of the eye-piece: when this is inserted in the straight cord, fixed to a nail on the wall and held by the person operating the machine, the eye-piece needs to be regulated so that the visual ray goes through the nail. In this way, when you look through the ocular it is as if your eye was positioned in the nail’s place.

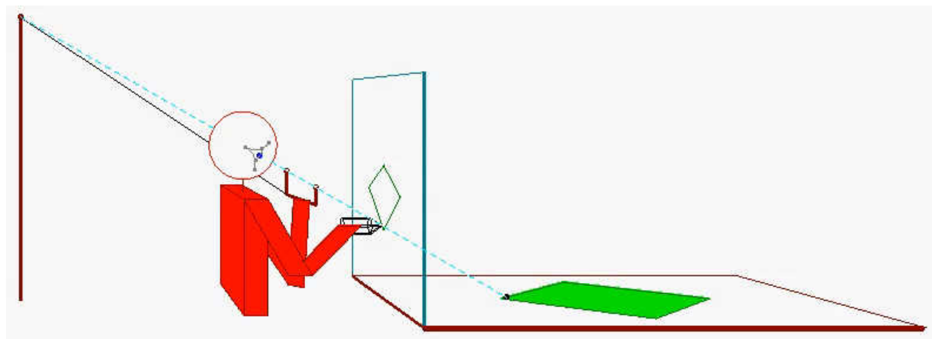
The FIGURE below shows the structure of this device and how it can be regulated.



Dürer goes on:

When everything is ready, follow these instructions. Set the object, the glass before the object, fix the cord to the nail O behind you, set the eye-piece so that the top is on the side of the glass and the hole on the side of O; hold the cord with your left hand, stretch it, move the eye-piece back and forth until it is at the correct distance as set by the glass, keep it still using your thumb and with your left hand rotate the small hole in the eye-piece towards your nose so that your right eye can see through the hole all the points of the object on the other side of the glass, keeping them at a line with the top of the visor. When you have finished positioning the sight with the straight cord, before your eye, take with your right hand a pen and, moving your sight, the top of the visor that you can see through the eye-piece will indicate the main lines and surfaces of the object that you will be able to draw with your right hand. The pen and the top of the visor will move at the same time. You will go very fast, as if you were copying something from one piece of paper onto another one.





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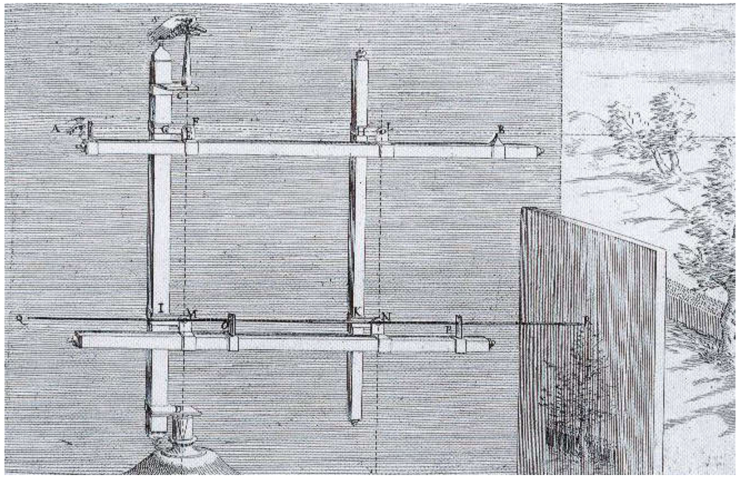
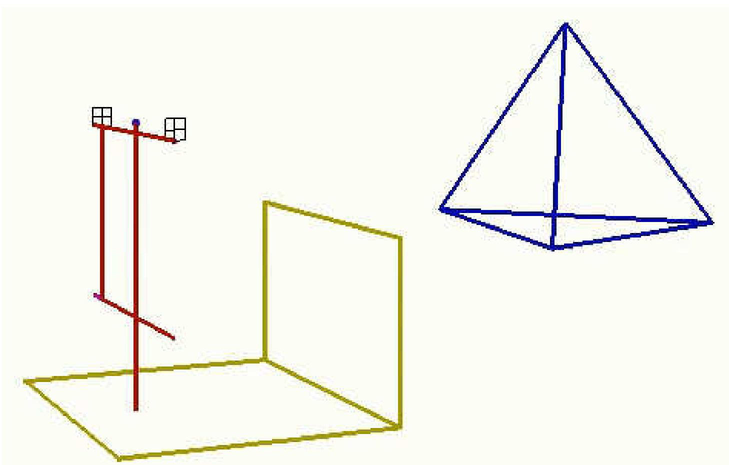
M.Bettini and C.Grienberger's Perspectograph

After concluding his critic to the false and useless perspectograph of Baldassarre Lanci, Danti ("Le due regole della Prospettiva Pratica di J. Barozzi…", 1583, Prima regola, Chapter three, p. 62) goes on:
People who would like to transform this instrument into the right one that could be useful: leaving the rules with the sight in the same place, make the base in the form of a square (instead of a circle) and take a plane board, instead of the circle (the cylindrical surface where the image was formed), and fasten a sheet of paper to it. Then follow the same instructions and everything will be correct. And even if this instrument does not make use of the string and everything has to be done through sights, it is a very good device; and because the board is fixed, we will not have any error, as sometimes happens with the doors that have to be opened and closed. The device described by Bettini (a Jesuit interested in the application of mathematical sciences and inclined to privilege magical effect) and constructed by Ch. Grienberger in 1635, puts into practice Danti's idea (see figure). This tool becomes particularly useful in topography (planimetry, maps). The extension of use from art to more practical uses is common to other instruments: the distribution of perspective devices is followed by the identification of their use as measuring tools. This contributes to addressing mathematicians' attention to perspective.

Cf. F. Camerota, "Nel segno di Masaccio", cat. Giunti 2001 (IX.1: La "terza regola"; IX.2: L'arte militare; IX.3: Una "nuova maniera di levar piante")





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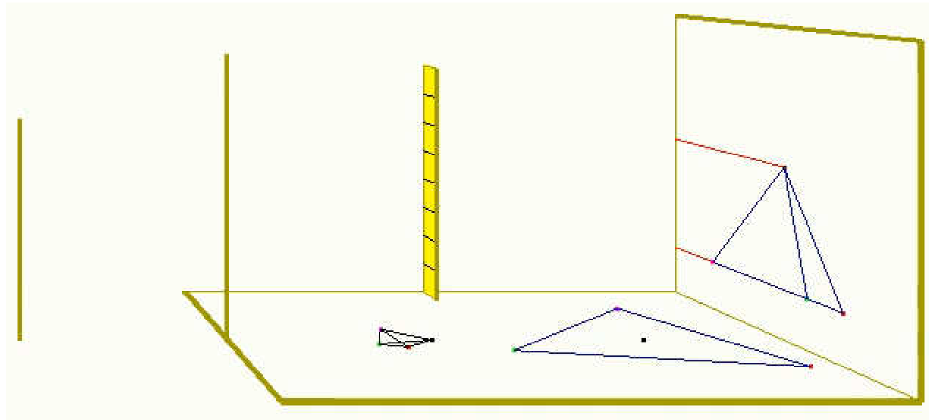
W.Jamnitzer's Perspectograph

It is well known that in order to construct the perspective image of an object following one of the classical rules, plan and elevation need to be known: these information are difficult to get from a real object. This is one of the reasons why mechanical perspective devices had a large diffusion. For example Danti writes ("Le due regole della Prospettiva pratica…", ed. Zannetti 1583, p. 57-58):
Despite my limited practice, I came to know how useful Albert's door is, because if we want to draw a perspective image of an object or building, even if we do the plan and enlarge it with extreme precision, we will never have a result as good as what we get with the door…. By using the rules we first need to do the plan of the object that we want to draw in Perspective and then reduce it…. This is very difficult and will never be as precise as with the door.
Relating to this, we can appreciate the singularity of Jamnitzer's invention, which requires, as the rules do, to know first the plan and the elevation. This way we can draw an object in perspective even without having the real object. Jamnitzer used his device to do the illustrations for "Perspectiva corporum regularium" (Norimberga, 1568). Below is a description from "Nel segno di Masaccio" (Giunti 2001, a cura di F. Camerota) that explains how the device works:
The viewer's eye is represented by the top of a pole placed at the back of the painter and a string going through there represents the visual ray. The endpoint of that string is tied to a cursor moving on a vertical rod, placed on any point of the base of the object to be drawn. The cursor is to be placed at the level of the object's height from the table. The perspective image of that point is identified by moving another vertical rod with a cursor so that it touches the string. In order to draw the point on the paper under the base of the object, the first rod and the base must be moved away and the second rod must be turned over to indicate the position of the cursor on paper.



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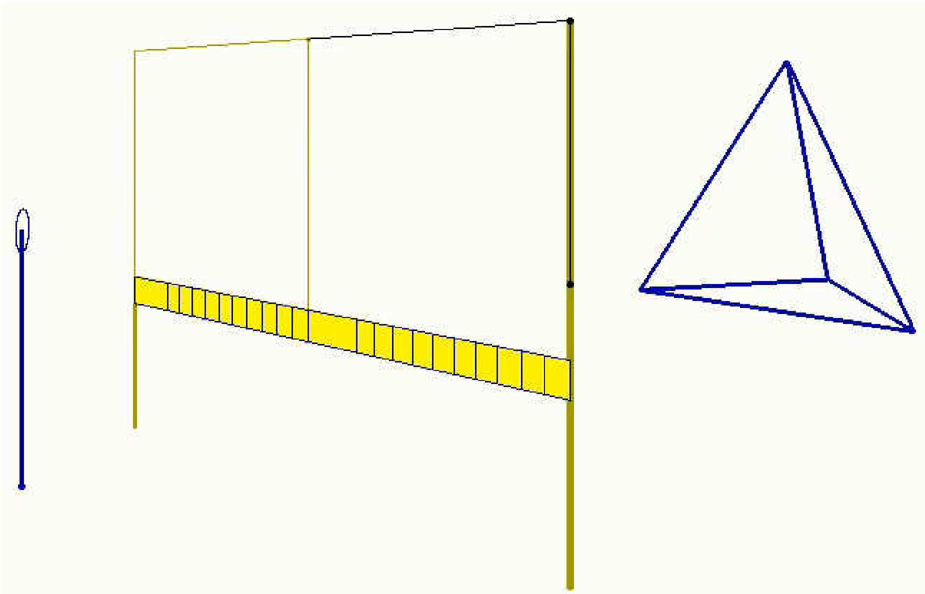
O.Trigini de' Marij's Perspectograph

We rely on Danti's description of this device. If this device did not require so much practice I would consider it excellent. M. Oratio Trigini de' Marij showed it to me … It is double, as you can see in the FIGURE AEFC, where BF replaces the door. Then you construct a rule GH that intersect both and you divide it into two parts GL and LH. Then you use a string IK and move the rule up and down so that intersections with the string are produced. The same technique is used to draw all the other points of the objects we want to put in perspective, following the instructions, as concerns distances and the rest, for the first door (Alebert's door).
This door requires experience: this is due to the fact that when the string touches GL, it will not always touch one of the divisions, sometimes it may be that it will be in the middle; therefore you will need lots of practice in order to be able to identify this same point in LH…



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Anamorphosis

A distorted projection or perspective requiring the viewer to occupy a specific vantage point, use special devices or both to view a recognizable image. Some of the media it is used in are painting, photography, sculpture and installation, toys, and film special effects. The word "anamorphosis" is derived from the Greek prefix ana , meaning "back" or "again", and the word morphe, meaning "shape" or "form". An oblique anamorphism is the visualization of a mathematical operation called an affine transformation. The process of extreme anamorphosis has been used by artists to disguise caricatures, erotic and scatological scenes, and other furtive images from a casual viewer, while revealing an undistorted image to the knowledgeable spectator.

Catottriche Anamorphosis Sphere

1. Introduction.
The catoptric anamorphoses are obtained by using planar or curved mirrors, while the optical anamorphoses follow the rules of perspective (but using a point of view that makes images deformed when observed from the front). The models in this exhibition represent some solutions to the following problem: you are given a mirror and you are required to trace figures on a planar surface such that their virtual images observed in the mirror represent given objects. Because we will look from a fixed position and with one eye only, we will need to take into account the relative positions of viewpoint, mirror and surface where the real image is formed. Real anamorphic images are generally deformed, confused, non easily interpretable. They get a familiar and harmonic aspect only if observed "per radium reflexum ex politis corporibus, planis, cylindricis, conicis, polyedris" (like virtual images). The mirror works as a decoder. In the 17th century these anamorphoses were mainly constructed "by eye", drawing on mathematical suggestions about how a rectangular grid is deformed through mirrors and optical or mechanical methods: experience and drawing ability were required. Nowadays it is possible to work with numerical calculations or geometric constructions. The help of a calculator is indispensable.

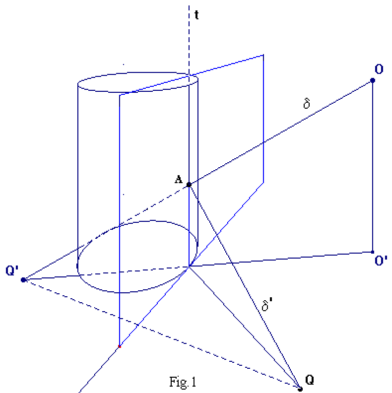
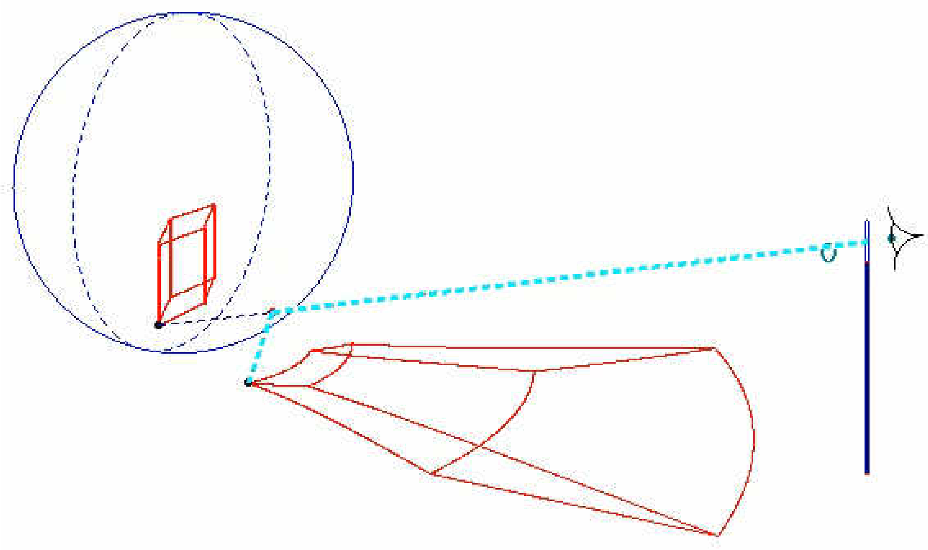
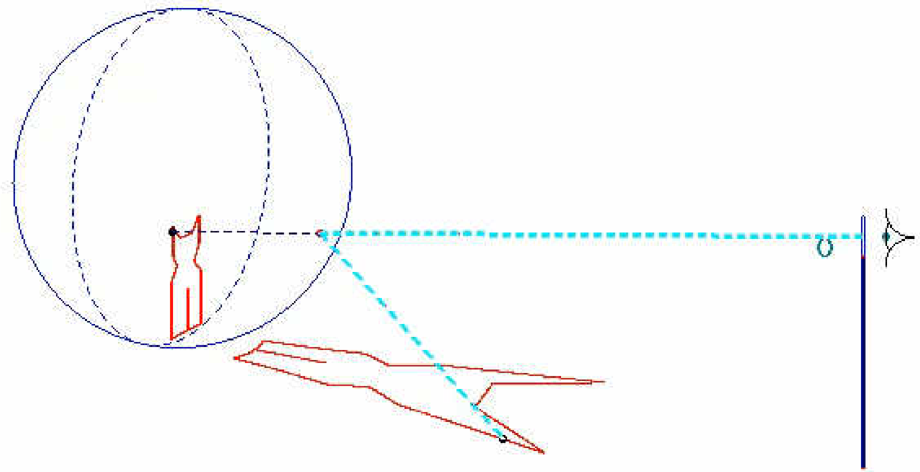


Fig.1

2. Observations about virtual images
A. Let us consider a planar mirror, leaning vertical or slightly oblique. An observer O (monocular) sees in the mirror a virtual image of the real space in which he is in. When the observer moves, changing his point of view, the reflected virtual objects (symmetrical image of the real objects with respect to the mirror) change in the same way (reciprocal dimension, position, distance) as the real objects. However, only the part of space internal to the cone with vertex in O' (symmetric image of O with respect to he mirror) and directrix in the contour of the mirror is visible.
B. Let Q be a real point (light source) in front of a cylindrical or conic mirror; the eye O sees Q' as the image of Q, where Q' is the symmetric point of Q with respect to the plane tangent to the mirror in A. Figure 1 illustrates the case of a cylinder. Hypothesis one: O stays fixed whereasQ generally changes position. The light rays coming from Q towards the eye O are incident in the neighborhood of a point B (A): the image of Q is Q (Q'), symmetrical of Q with respect to the plane tangent to the mirror in B along the generatrix t* (t) of the cylindrical surface. Because an extended object is a set of points Q, then: the correspondence between each point of and each point of its virtual image ' is not a symmetry with respect to a fixed plane, but it is a symmetry with respect to a plane that, in the passage from a point of to another one, changes position; therefore the image ' seen by O is, with respect to , deformed. Second hypothesis: Q stays fixed, point O changes his position. Also in this case, the light rays coming from Q towards the eye O are incident in the neighborhood of a point B (A): because the plane tangent to the mirror in B does not coincide with the tangent plane in A, the image of Q is a point (Q'') different from Q'. B non coincide con quello tangente in A, l'immagine di Q sarà un punto (Q") diverso da Q'. Because an extended object is a set of points Q, then the cylindrical mirror gives a virtual (deformed) image ' depending on the position of O at each instant.*





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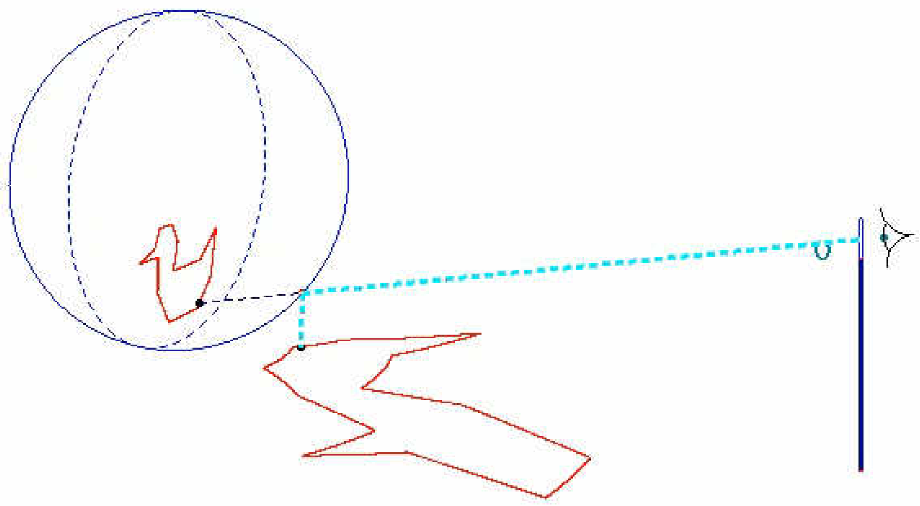

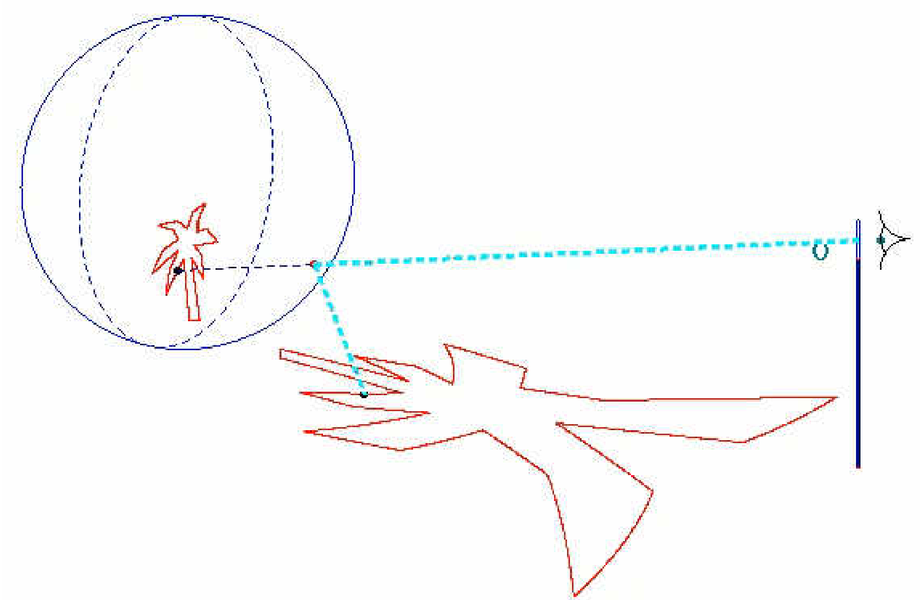

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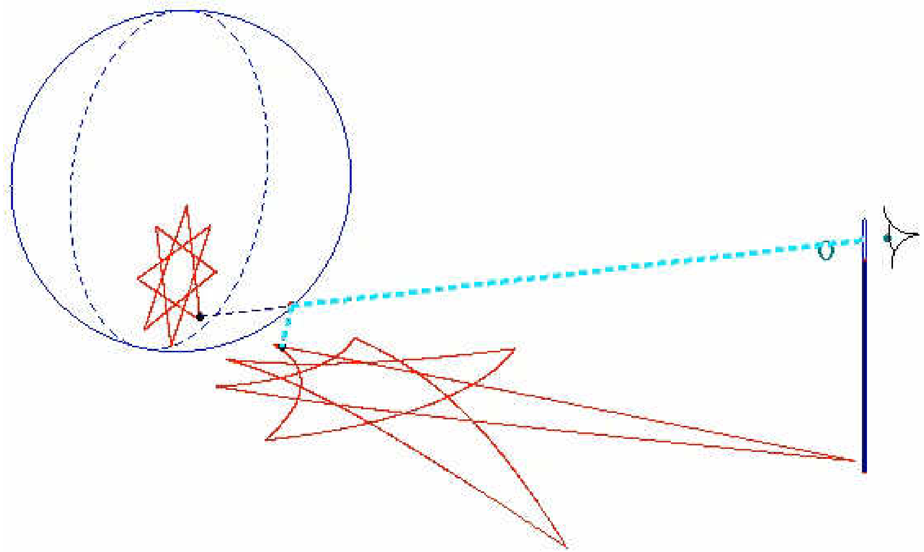
Catottriche Anamorphosis Sphere



Anamorphosis

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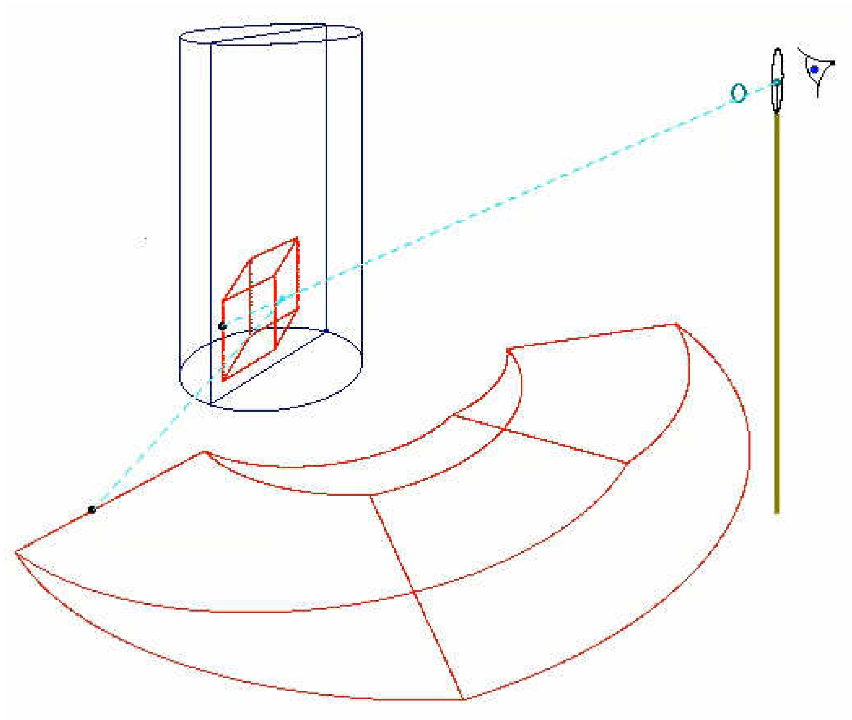
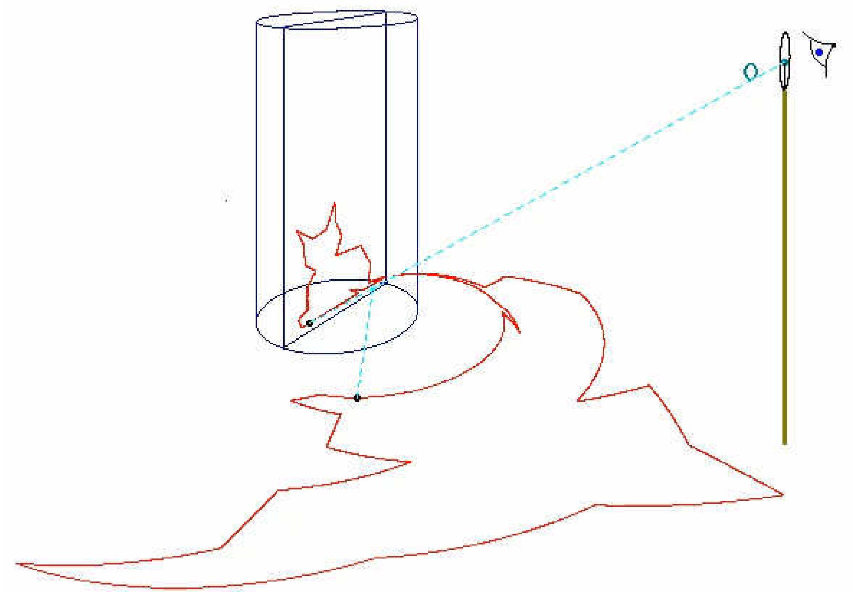
Catottriche Anamorphosis Cylinder





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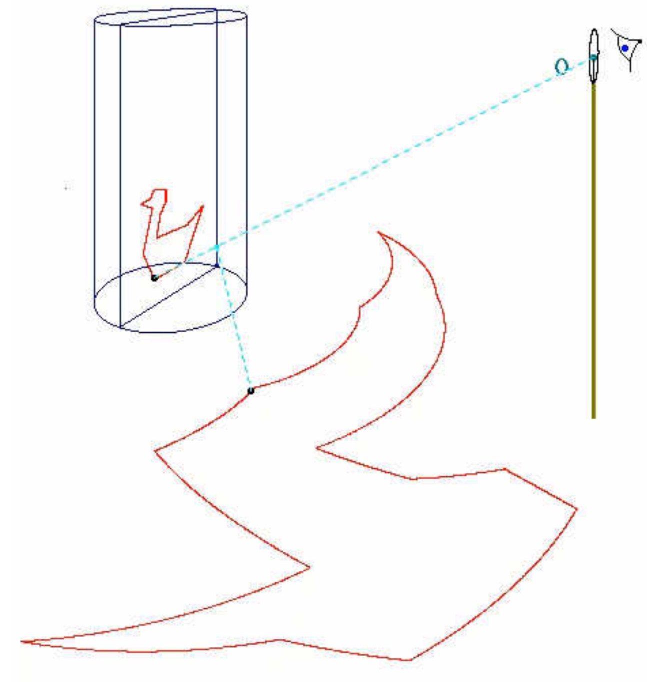

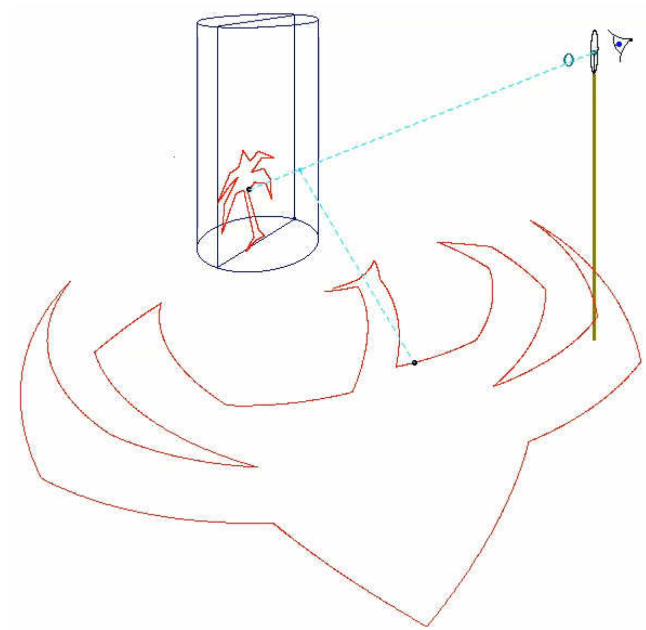

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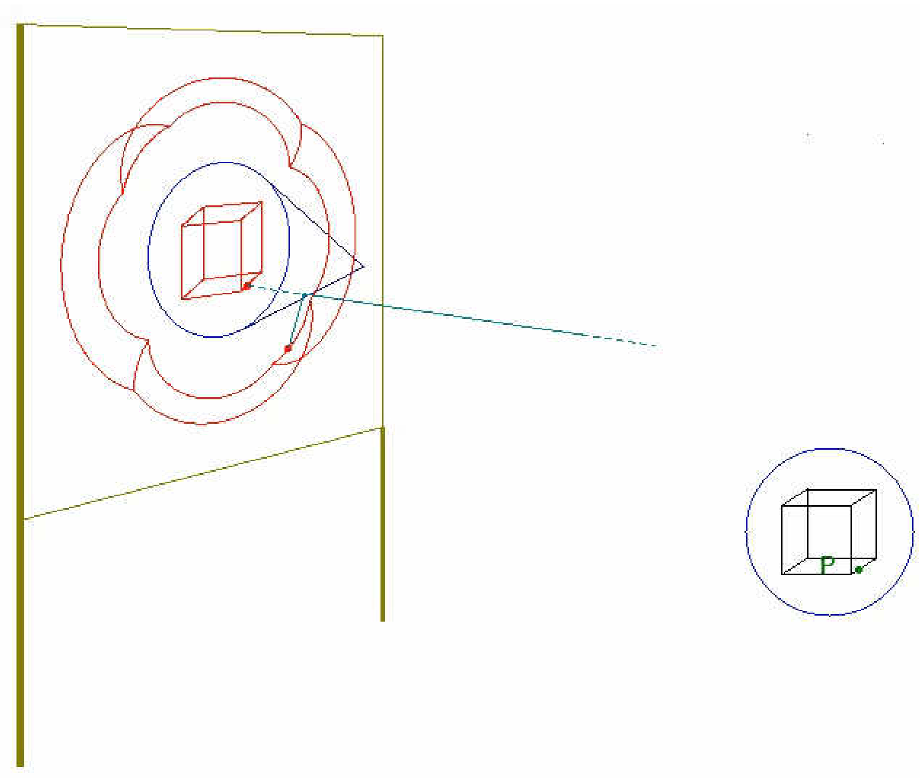
Catottriche Anamorphosis Cone



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Optical Anamorphosis (Niceron)

In "Prospettiva pratica", Ludovico Cardi (known as Cigoli) observes that if a young painter needs to draw a view of streets and buildings or some other composition of objects placed in different positions with respect to one another or something complex like the human body, he may come to believe that geometric rules cannot be applied in that case and therefore decide to draw by eye; but such a decision would contradict the noble art of painting (1). The need to face such difficulty leads to the definition of a third rule based on a methodical and aware use of tools. (2) Cigoli (an expert in the representation of far away objects – remember the representation of the Moon seen from a telescope) shows dissatisfaction towards traditional devices (like the door) that are thought to be imprecise and imperfect. He meant to invent a machine that was flexible and comfortable to use but also very precise, at the level of the requirements of the new sciences of his friend Galileo. The result of his efforts was an 'automatic' instrument that remained a reference point for the more advanced tools until the first half of the 19th century. When describing the construction and use of his device, Cigoli gives detailed instructions, with figures of the components of the device, for people who may want to construct and use one themselves. The device is made of two vertical elements fixed on a horizontal structure screw to a base. One of these elements is articulated so that its height and the position of the sight placed at the top can be changed, whereas the other element moves on a horizontal rod which functions as the base of the intersection. A string with a small ball at the end moves along the second vertical rod going through two pulleys and along a horizontal rod up to the handle of a 'tracer' …. The person drawing uses his left hand to move the vertical rod right and left through the plane of intersection and his right hand on the 'tracer' to move the ball up and down along the vertical rod. The position of the ball, seen through the sight, can be moved continuously on the contour of the object to draw while the drawer with his right hand traces the corresponding configuration on the sheet of paper fixed on the base. (3) Even if this device is not at all easy to use, it is the first one that incorporates an 'automated' system for drawing: only one operation is needed in order to represent an object directly on the drawing sheet, as it appears on the plane of intersection. (4) Cigoli's manuscript was not published, however his perspectograph had a good diffusion: François Niceron testifies he found one model in Paris, in the house of L. Hesselin, one of Luigi XIII's counsellors. Padre Jean François Niceron, belonging to the Ordine dei Minimi, friend of Mersenne, Mathematics teacher at Trinità dei Monti, theologian, expert in perspective (particularly interested in anamorphoses) and in physics, auxiliary visitor of convents belonging to his order (therefore often travelling from Italy to France), was born in Paris in 1613 and died in Provence in 1646. The book that made him famous is "La Perspective Curieuse", written in 1638, and published in Latin in 1646 with the title "Thaumaturgus Opticus". In this book Niceron gives a very detailed description of a Scenographum Catholicum (Universal machine for scenography) which coincides with Cigoli's perspectograph. Niceron says that he used to go to the house of the powerful politician Ludovico Hesselin in Paris. Niceron describes this house as luxurious, situated in the eastern part of the city, by the river: it had a beautiful garden, great view, fresh air, and elegant architecture. The rooms inside included a large library with all sorts of old and new volumes. It was in this house that Niceron came across the pieces of Cigoli's perspectograph (in a portable version): it had been sent to Hesselin from Italy, disassembled. After trying to put it together, Hesselin gave it to Niceron, confident in his competence regarding optics and the theory of projections. Niceron managed to assemble it correctly and showed how it could be used in Scenographicus delineationibus. It is a very versatile instrument. By bending the vertical rod we can obtain the projection of forms on inclined planes (therefore it is useful for illusionistic decorations and for anamorphoses). It can also be used in reverse, to project a perspective image on any curved or plane surface.

(1) Cf. M. Kemp, La scienza dell'arte, Cap. IV, pag. 198
(2) Cf. F. Camerota, La terza regola, in Nel segno di Masaccio, IX.1, p.190
(3) Cf. M Kemp, op. cit., Cap. IV, p. 199
(4) Cf. M Kemp, op. cit., Cap. IV, p. 200

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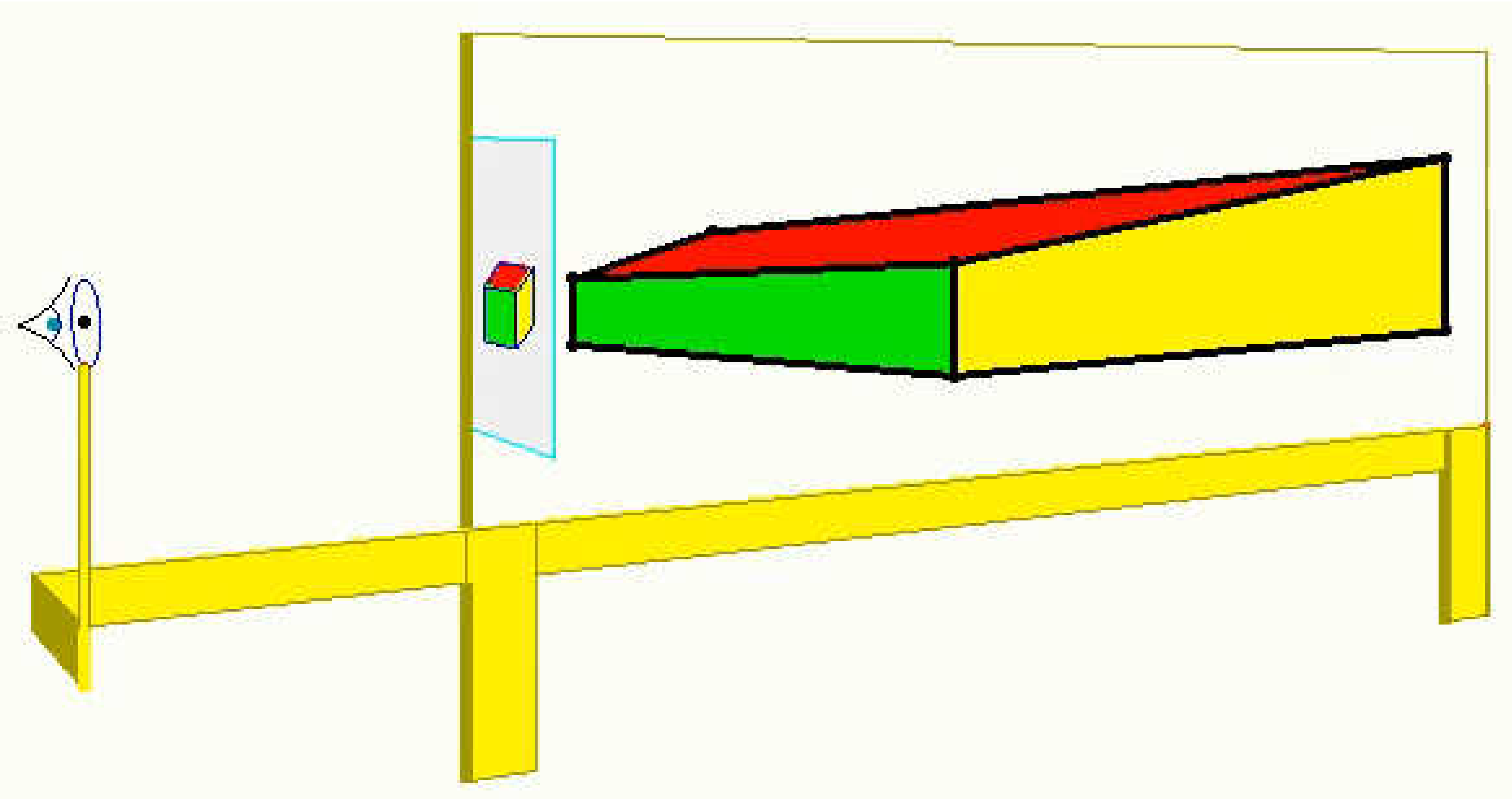
Optical Anamorphosis (Niceron)



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Perspective & Transformations

This section briefly describes how the theory of transformations (developed in the 19th century) is rooted in the studies on perspective.

1. In the initial "creative" period (Brunelleschi, Masaccio, Alberti, Piero della Francesca…) it was common belief that linear perspective, considered a geometric science, was the foundation of arts. The very close relationship between artists' theoretical research and practical activity went through a period of crisis: the communication between theorists and practitioners became more difficult and at some point got to an end. A number of reasons may explain the increasing separation between scientists and practitioners. Inevitably, conflicts between the needs of the invention practices and the constraints of the geometric rules arose (Michelangelo sustained that an artist must have the sixts in his eyes, rather than following mathematical procedures) [1]. Printed volumes on perspective appeared in Italy quite late, only towards the last part of the 16th century (the circulation of manuscripts was very limited): the practices carried out by craftsmen were disseminated more easily than the new methods developed by geometers. In some cases the influence of the Counter-Reformation was manifested in attention being paid more to content than to technical virtuosity. Moreover, in the illustration of their books many scientists never managed to obtain the quality and the expressive ability of the engravers who illustrated simpler texts for empirical use. However, the key element is of a different nature: the problem of perspective as the science of art was strictly linked to the problem of plane representation of threedimensional objects. This matter was considered important in many different activities: for example in cartographic projections (representation of the earth, the sky, the sun, the moon) or in the management of military fortifications, hydraulic constructions, cathedrals and palaces. As a consequence, scholars developed perspective geometry beyond the needs and comprehension of artists, transforming it into an abstract theory of projections. The writing of treatises and manuals for perspective shifted from the hands of the inventors, that is artists with sympathy for science, to the hands of codifiers, that is scientists who rarely paid attention to the needs of art.[2]

2. In the 16th century we can observe an alternation of 'simple' books, taking empiricism to the extreme, and 'difficult' books, taking abstraction to the extreme and being incomprehensible for non-initiated people[3]. Not many equilibrated books can be found: with this respect one of the best treatises is Commentari sulle due Regole della Prospettiva pratica written by J. Barozzi (architect) together with E. Danti (mathematician and cartographer).[4] This is an illustrated book aimed to dissemination; it is very good from the didactic point of view and at the same time very rigorous. On the other hand there are volumes like the one by Commandino, Benedetti, Guidubaldo del Monte[5] that are very difficult to read; however with them we can see the birth of projective geometry as an autonomous discipline: still related to the science of painters, but more and more detached from that as far as means and scopes[6]. These books constitute the basis for future interactions between perspective and other fields in which mathematics and geometry were in the process of evolving and developing new methods. Some related information are included below:

a) The treatise by Commandino (1558) was titled "Commentario al Planisfero di Tolomeo e di Giordano Nemorario": it includes what will be called stereographic projection of the celestial sphere (therefore it is aimed to an audience of astronomers and mathematicians). The third part includes a general theory of projections, that develops original methods. In particular he studied the perspective image of a circle when the viewpoint changes (the image is either a circle or a conic): he uses the concept of triangle for the axis and a particular position of that triangle with respect to the diameter of the circle, therefore it is not yet a general method (he considers the cone with vertex in the viewpoint and projecting the circle). (Some years later Commandino examined also the projection of a circle with parallel rays).

b) The treatise written by Benedetti (De rationibus operationum Perspectivae) (1580), very short (only 22 pages), is inserted in a volume of miscellaneous mathematics put together with the intention of rectifying some empirical procedures still used by artists. However his constructions are very abstract: he does not consider the physical process of vision, and he does not take into account the needs of painters.[7]

c) When Guidubaldo Del Monte published in 1600 his treatise on Perspective, he had already investigated the problem of orthographic projections and proven that the sections of a cylinder are ellipses[8]. In Perspectivae Libri Sex, he defines the vanishing point for any pencil of parallel lines and the horizon line as the set of vanishing points; he uses a movement of the frame onto the ground plane which will be the basis of the birth of the concept of homology; he gives a beautiful construction for the perspective image of a triangle (cf. Scheda OBLIQUE PERSEPCTIVE, HOMOLOGY (1)) in which one of the fundamental theorems of projective geometry is used and will be explicitly formulated by Desargues in 1639; he uses the same method to obtain the perspective image of a circle. Moreover, he presents a complete theory of shadows 9.

(cf. Scheda OBLIQUE PERSEPCTIVE, HOMOLOGY (1)) in which one of the fundamental theorems of projective geometry is used and will be explicitly formulated by Desargues in 1639; he uses the same method to obtain the perspective image of a circle. Moreover, he presents a complete theory of shadows [9].

3. In the 17th and 18th century, experimental work included strange and bizarre applications, as for example anamorphoses, scene-painting, illusionistic representations. A new space was opened up to theorists too, while the intersection, but at the same time also the distinction, of focus between rigorous treatises and operative manuals (with no care for proofs), together with textbooks with didactical aims, remained. Italy lost the predominance it had in the 16th century and the work moves to the Netherlands and France at the beginning and then to England and to the rest of Europe. Throughout this period, the theory of projections absorbed the methods that had opened up new spaces for mathematical thinking, unifying the use of algebra and motion. The works by Stevin, De la Hire and Lambert contained a first hint to the concept of homology. The works of Desargues and Pascal[10] included the theory of conics within the mathematics of projections and introduced new concepts and proving techniques, which can be found again in Newton and Jaquier's studies on the shadows of planar curves[11], in a more complete form and based on adequate analytical apparatuses. Themes and problems emerged and became source of comparison for analytical and synthetic methods: the theory of projections (now completely independent from the need for representing reality and imitating the natural world) gave birth to new concepts and new proving techniques and the equations of homologies (at the beginning as a tool to transform curves) appeared. Finally, the links with technical systems (e.g. assonometry) became stronger. All results will be reorganized and codified in the works of Monge and Poncelet, within two new sciences (descriptive and projective geometry). The theory of transformations of the 19th century is based on this first target.

1.Cf. G.P. Lomazzo, Trattato dell'arte della Pittura, scultura e architettura, Milano 1584; Idea del tempio della pittura, Milano 1590, in R. P. Ciardi, Scritti sull'arte, Firenze 1973.

2. Cf. L. Vagnetti, De Naturali et Artificiali Perspectiva, Firenza 1979, pag. 290

3 .Cf. L. Vagnetti, op. cit., ibidem.

4. J. Barozzi, Le due regole della Prospettiva pratica di M.J.B. da Vignola con i commentari del R.P.M. Egnatio Danti dell'ordine dei Predicatori, Matematico nello studio di Bologna, Roma 1583.

5. F. Commandino, Ptolomaei Planisphaerium, Jordani Planisphaerium, Federici Commandini Urbinatis in Planisphaerium commentarius, in quo universa Scenografices ratio quam brevissime traditur, ac demonstrationibus confirmatur, Venezia 1558; G. B. Benedetti, De rationibus operationum perspectivae, in Diversarum speculationum mathematicarum et Philosophicarum liber, Torino 1580; G. Del Monte, Guidi Ubaldi e Marchionibus Montis perspectivae Libri sex, Pesaro 1600.

6 .M. Kemp, La scienza dell'Arte, Firenze 1994, p. 99

7 .M. Kemp, op. cit., pag. 101; cf. anche L. Vagnetti, op. cit., pag. 303.

8. The ortographic projections of circles are ellipses, however at the time of G. Del Monte they were still considered "strange curves" and not conics.

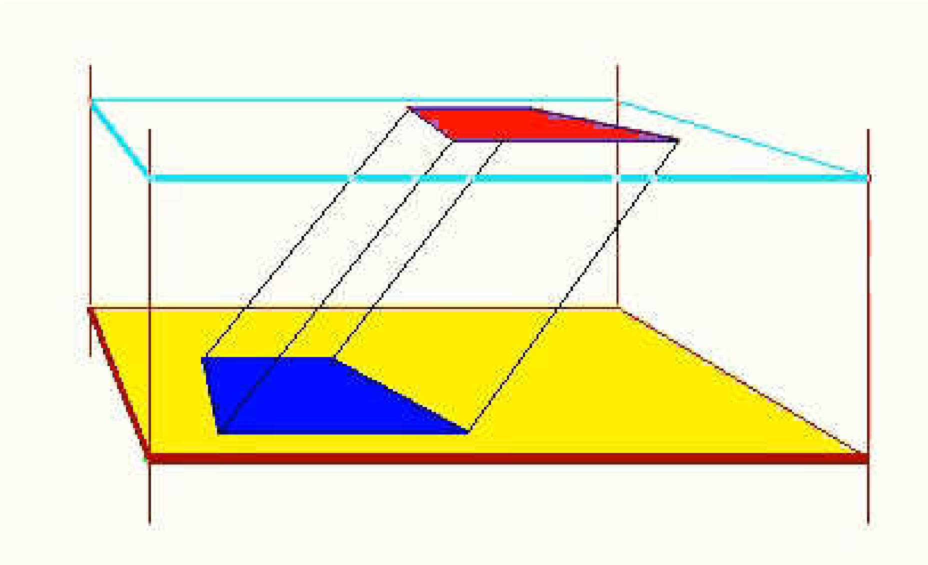
9. Cf. M. Kemp, op. cit., p. 106.

10. G. Desargues, Exemple de l'une des manières universelles du S. G. D. L. touchant la pratique del parspective sans empoyer aucun tiers point de distance ny d'autre nature, qui soit hors du champ de l'ouvrage, Parigi 1636; Brouillon project d'une exemple d'une manière universelle du S. G. D. L. touchant la pratique du trait à preuves pour la coupe des pierres en l'Architecture, Parigi 1640; Brouillon Project d'une atteinte aux evénements des rencontres d'un cone avec un plan, Parigi 1639. B. Pascal, Essai pour les Coniques, Parigi 1640. De La Hire, Nouvelle Methode en géometrie pour les Sections des Superficies coniques et Cylindriques qui ont pour bases des Cercles, ou des Paraboles, des Elipses et des Hyperboles, Parigi 1673; Nouveaux élemens des Sections Coniques, les lieux gèometriques, la constrution ou effection des Equations, Parigi 1679; Sectiones Conicae, 1685.

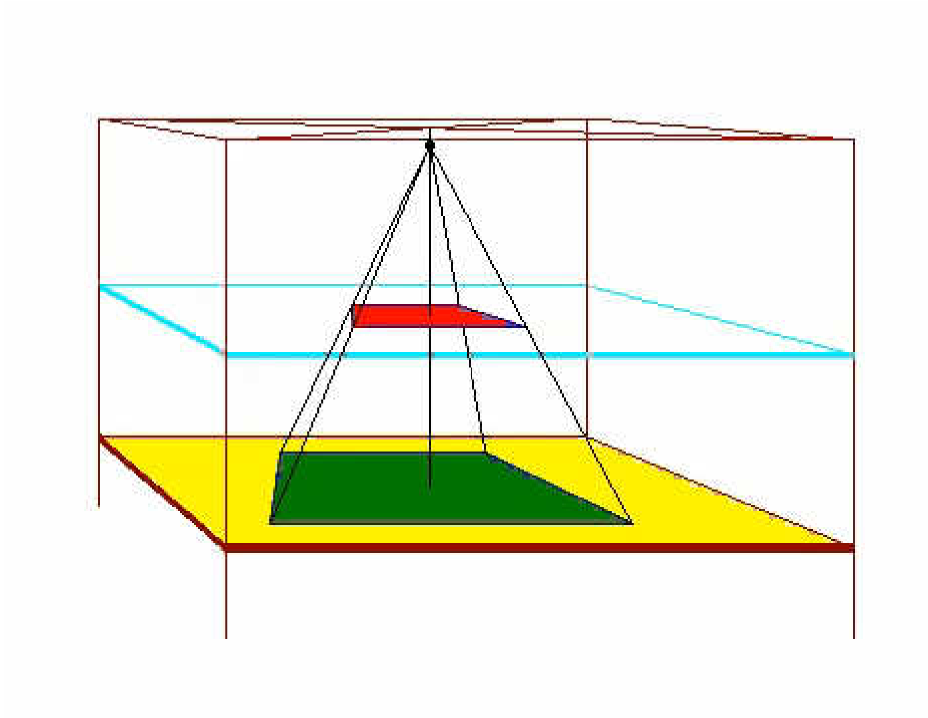
11. There is a close link between De La Hire's method to determine the perspective image of a point and Newton's method illustrated in Principi Matematici della Filosofia Naturale. The existence of this relationship is noticed for the first time by M. Chasles (Aperçu Historique, 1837).

(1.)Cf. A. Comessatti, Geometria descrittiva e applicazioni, in Enciclopedia delle Matematiche Elementari, Hoepli 1964.

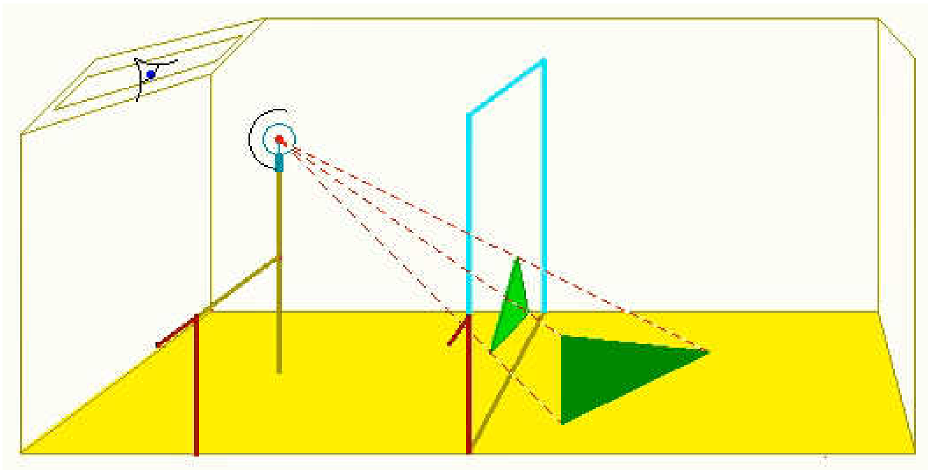
Genesis of Translation



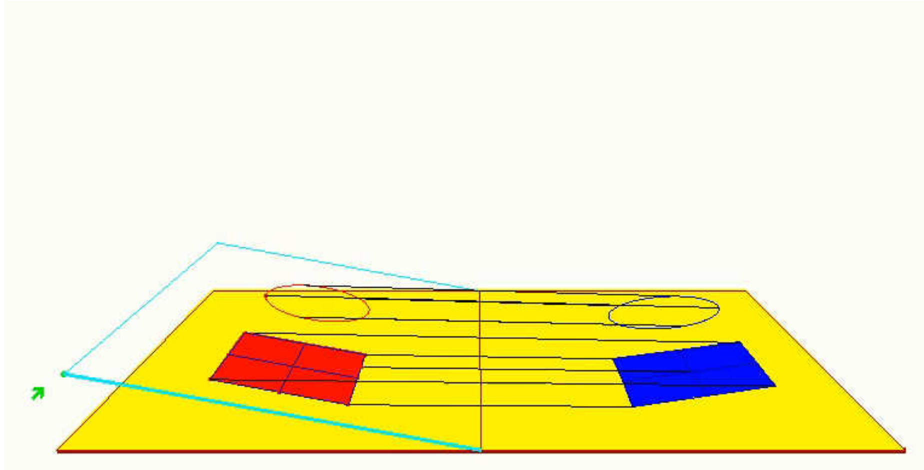
Genesis of Homoteties



Images of Lines Parallel to the Frame



Genesis of Affine Transformations



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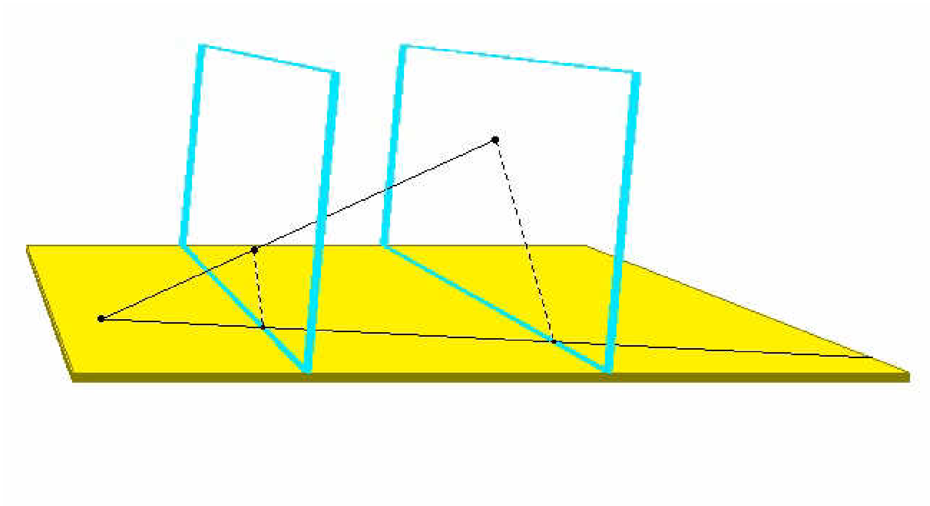
De la Hire’s Method



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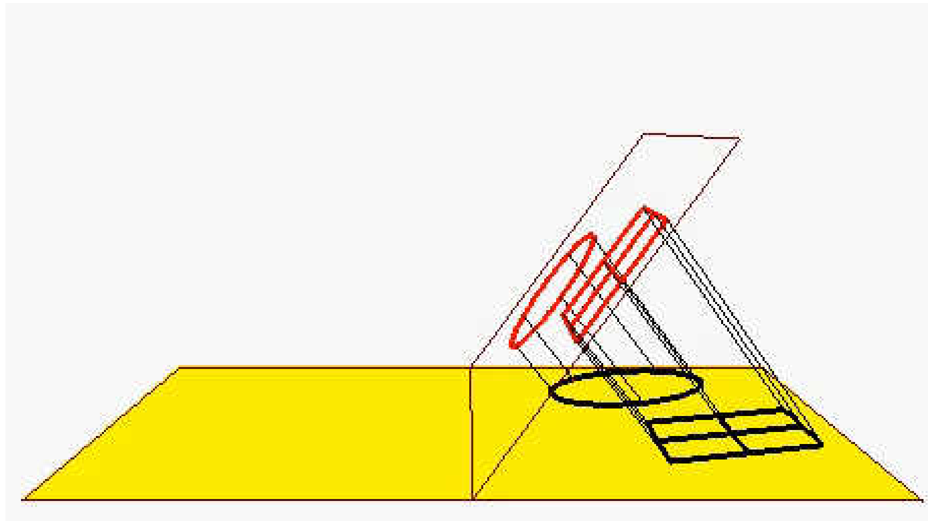
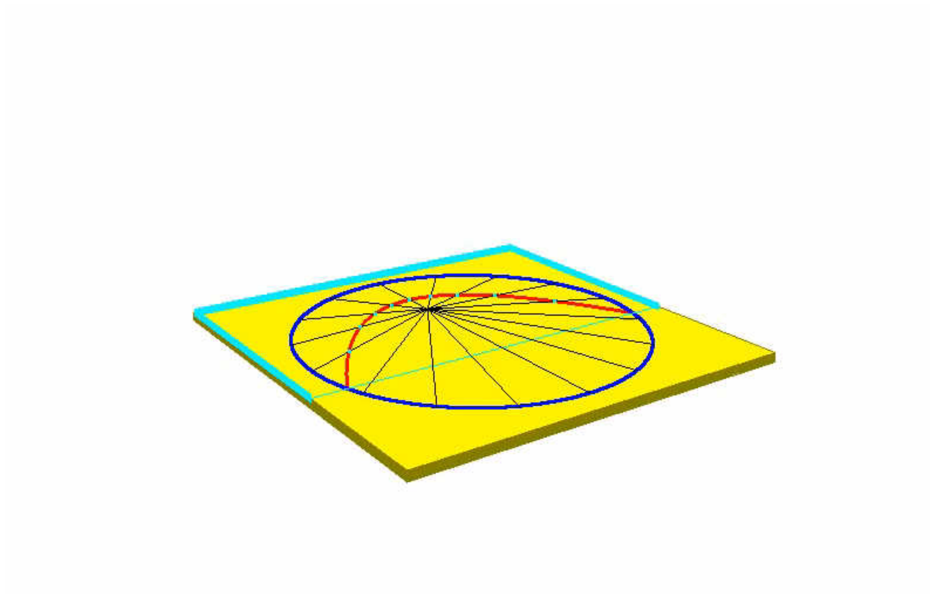


De la Hire’s Method 2



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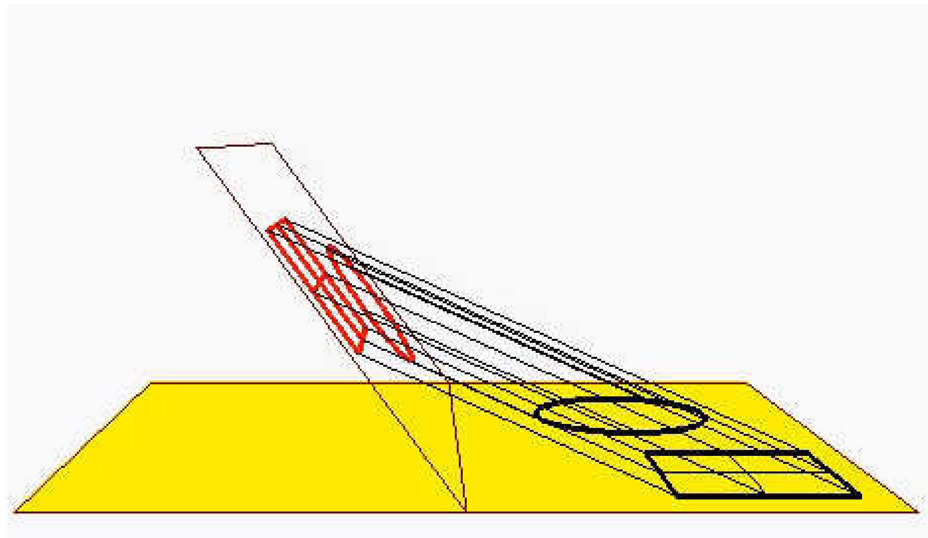

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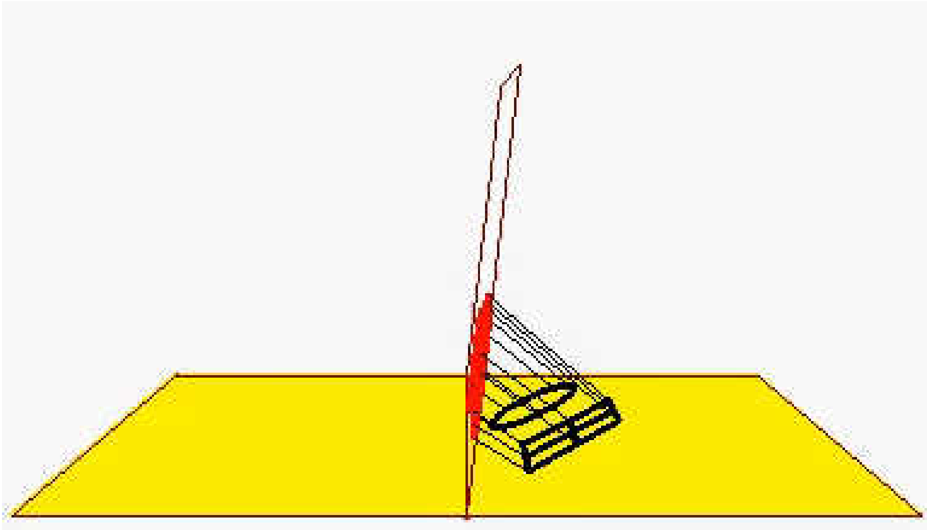

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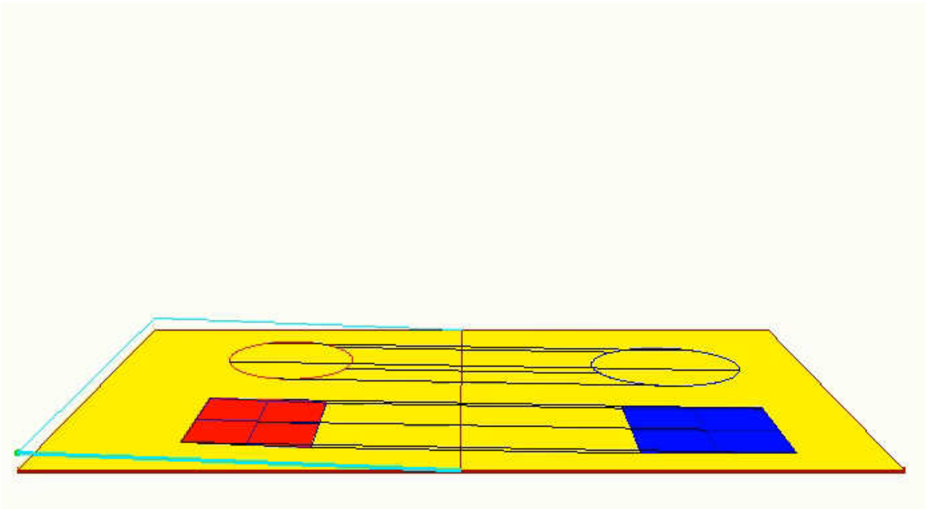

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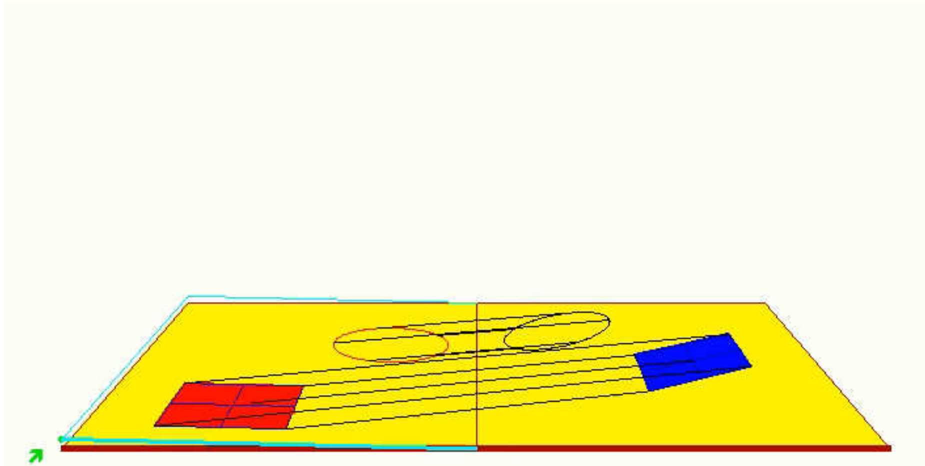
Genesis of Affine Transformations 5



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Projections & Pantographs

From Greek roots παντ- "all, every" and γραφ- "to write", from their original use for copying writing is a mechanical linkage connected in a manner based on parallelograms so that the movement of one pen, in tracing an image, produces identical movements in a second pen. If a line drawing is traced by the first point, an identical, enlarged, or miniaturized copy will be drawn by a pen fixed to the other. Using the same principle, different kinds of pantographs are used for other forms of duplication in areas such as sculpture, minting, engraving, and milling.

Because of the shape of the original device, a pantograph also refers to a kind of structure that can compress or extend like an accordion, forming a characteristic rhomboidal pattern. This can be found in extension arms for wall-mounted mirrors, temporary fences, scissor lifts, and other scissor mechanisms such as the pantograph used on electric locomotives and trams.

Shadows and Perspective

The fact that a same mathematical model describes both the formation of perspective images and the formation of shadows is obvious for us but it is the result of a long historical process. 1. The existence of a link between the study of shadows and the study of perspective was recognised experimentally through experimental observations (probably made by the constructors of solar watches). For example, some observations conained in L.B. Alberti's manuscript "De Statua", a famous drawing by Leonardo (Figure 1) (2) and many other clues reveal an awareness of the fact that the contour we perceive every time we observe an object is the same as the contour of its shadow, provided that the shadow is produced by a light placed in the same place as the viewer's eye. (3) This discovery shows that the production of shadows belongs to the empirical foundation of perspective.

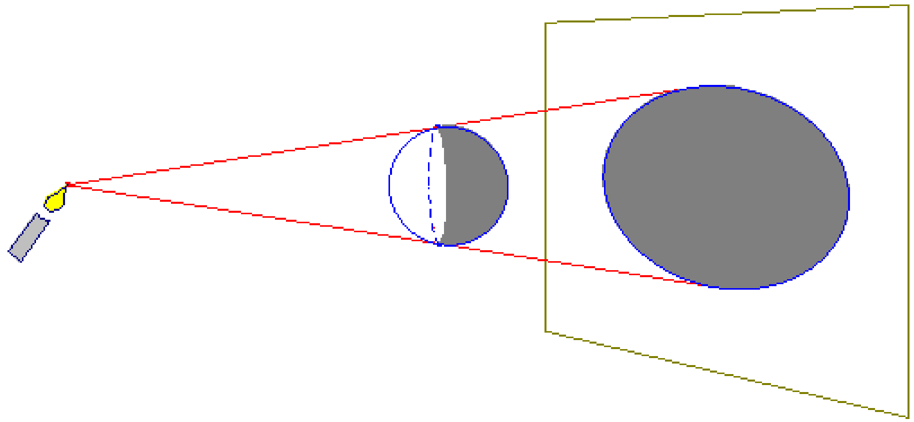
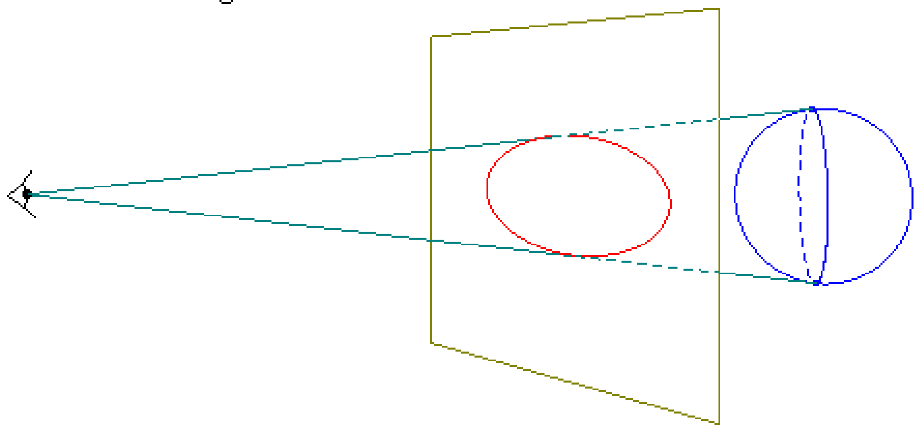
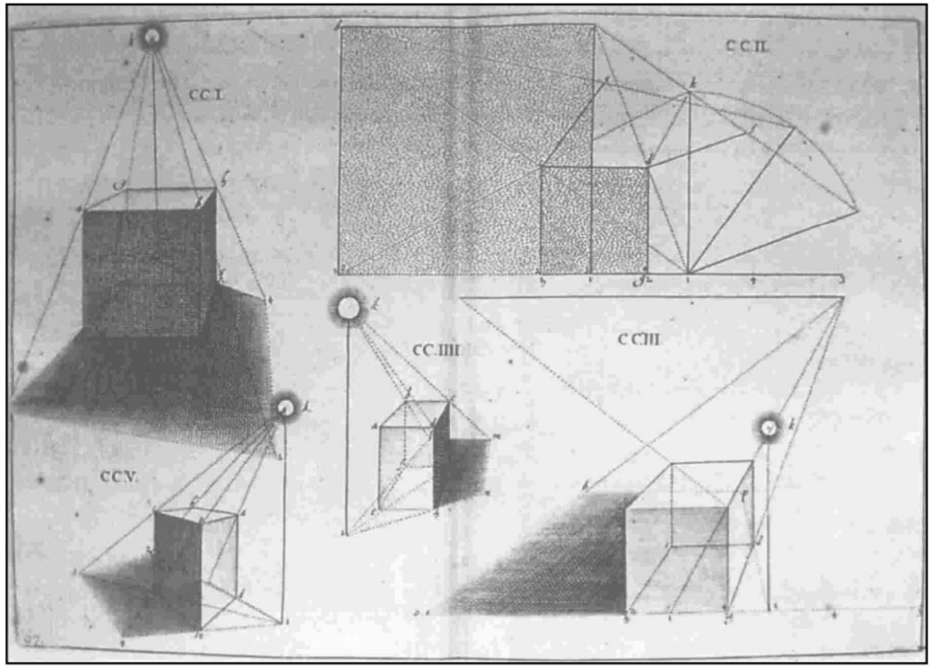


Fig. 1



2. However, since the Renaissance culture was anthropocentric, at the time it was difficult to distinguish the presence of a projection (shadows) within another projection (the perspectiva artificialis), that is the coexistence of two points of view: the viewer's eye and the source of light. In order to do that it was necessary to consider the problem of shadows as something external to the configuration, not linked to the problem of the intersection of the visual pyramid and the image plane(4). Before the beginning of the research strand leading to the discovery of exact geometric methods for the construction of shadows (to which all scholars dealing with perspective collaborated) used by painters, this "epistemological obstacle" needed to be overcome. The explicit observation that if the viewer's eye and light source coincided then shadows could not be observed can be found in Galilei (1611) and Accolti (1624) (5).

The research about the construction of shadows involved all main scholars in the field of perspective. Differentiating rigorously natural and artificial, pointwise and non pointwise sources, in relation to shadow effects, was not an easy task. Both artists (like Alberti, Leonardo, Dürer, etc) and theorists (like Barbaro, Marolois, D'Aguilon, etc) implicitly attributed the same geometrical typology to shadows produced by the sun, the moon and other light sources of the same intensity. For example, in all three drawings in Figure 2 (taken from the treatise on Perspective by François d'Aguilon - 1613) (7) we can see a representation of the sun: this clearly indicates the uncertainty about the distance and the nature of the light source (8). In the same way as the viewpoint in perspec-tive drawings, the light source in projections was almost always con-sidered to be at a finite distance. A rigorous discussion of shadows produced by a light source at a finite di-stance can be found in Libro Quinto by G. Del Monte (Guidi Ubaldi e Marchionibus Montis Perspectivae Libri sex, Pesaro 1600).



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Projection & Pantographs

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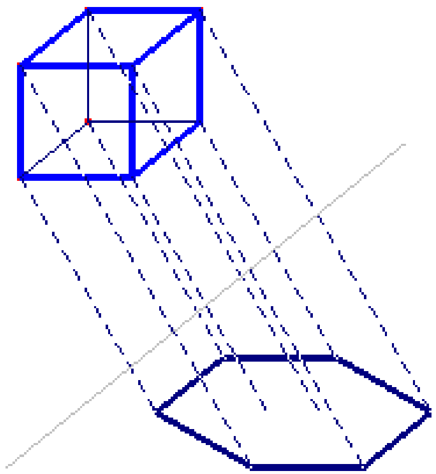
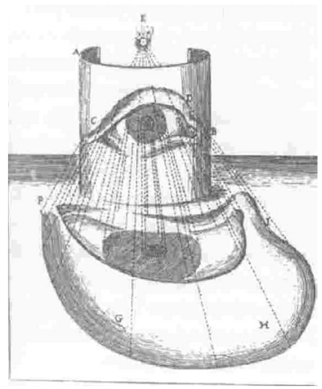
4. Of course it was well known that as the light source was taken further away then the light rays tended to become parallel; parallel projections were used to draw perspective representations and representations of tri-dimensional figures. However, the relationship with perspective was not yet clear: an importantcontribution to the solution of this problem is to be found in the links between the science of perspective and the techniques for cartographic projections, astronomical measurements and similar procedures to carry out measurements on the earth and the cosmos. In the second half of the 16th century, Gemma Frisius and Juan De Rojas (drawing on the work of Tolomeo and examining orthographic projections (Scheda)) explicitly state that the method of parallel projection could be deduced from perspective and introduced the concept of "eye at an infinite distance" (9). Despite the doubts that Guidubaldo Del Monte had(10), the ideas of Frisius and De Rojas becam universally accepted and eventually found a place
within projective geometry.

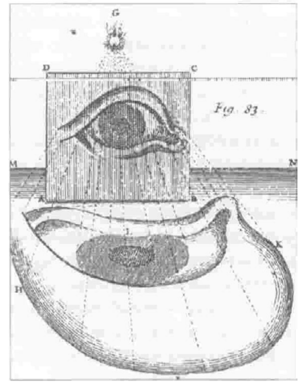
5. The studies on planispheres, celestial and terrestrial maps, solar watches and astrolabes had a very important role in the development of geometry. They contributed to bringing perspective closer to pure science and mathematics, making it more abstract and less subordinated to the specific problems of painting: perspective gradually became a theory of projections(11). From the second half of the 15th century onwards, many treatises around perspective are less and less aimed to practitioners and illustrate very general procedures and explicitly (from the language point of view and from the kind of questions they tackled) address an audience of geometry (or science) experts. Regarding our topic, we mention the comment of Federico Commandino to the Tolomeo's Planisphere, which can be considered a short treatise about perspective (12). His constructions of a perspective image (on a frame perpendicular to the ground) can be used with a figure that is placed on either sides of the frame (traditional perspective or case of a shadow projected from a light source) or across the frame (generic central projection with no immediate physical interpretation). He also proves that under particular circumstances the perspective image of a circle can be a circle (besides any conic): this fact is very relevant form a geometric point of view but certainly not very significant from a practical point of view (13).

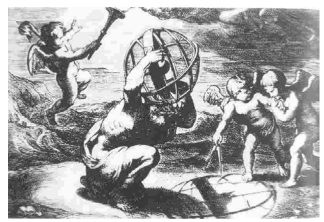
6. Parallel projections almost perfectly explain the nature and behaviour of sun shadows, but cannot interpret rigorously the phenomenon of vision because they require a visual pyramid with improper vertex. Despite that, the images produced through parallel projections may have sufficient perspective expressiveness (they are still used in many geometry manuals to represent polyhedrons or 3D objects). Moreover, parallel perspective leads to the determination of assonometric systems that have been largely used in technical drawing and military art. François D'Aguilon (1613) (14) was the first person to see the importance of orthographic projections in the representation of the earth and the celestial sphere and in the representation of buildings and any other thing. Figure 3 is the assonometric projection of a cube and the construction Fig.3(through parallel rays) of one of its possible sun shadows.

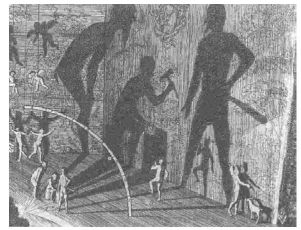


7. As far as the production of shadows, the separation between theory and practice is stronger than in the case of the historical evolution of artificial perspective. Even if the theoretical foundation was incomplete (or we may judge it as incomplete), still this did not affect an effective production of shadows, with a variety of different uses. There were many "workshop practices" which allowed inserting realistic shadows of people and objects in paintings, even if they were not accurate from a geometrical point of view. Theatres of shadows were quite popular (Figure 6) (18). The projection of grids by means of sources of light was used to paint illusionistic images on ceilings or vaults (Andrea Pozzo was a key person in this field), to construct anamorphoses (Figure 4 and 4a) (16) and to draw grids on astronomic or geographical maps and curves that was impossible to obtain in other ways (Figure 5) (17).



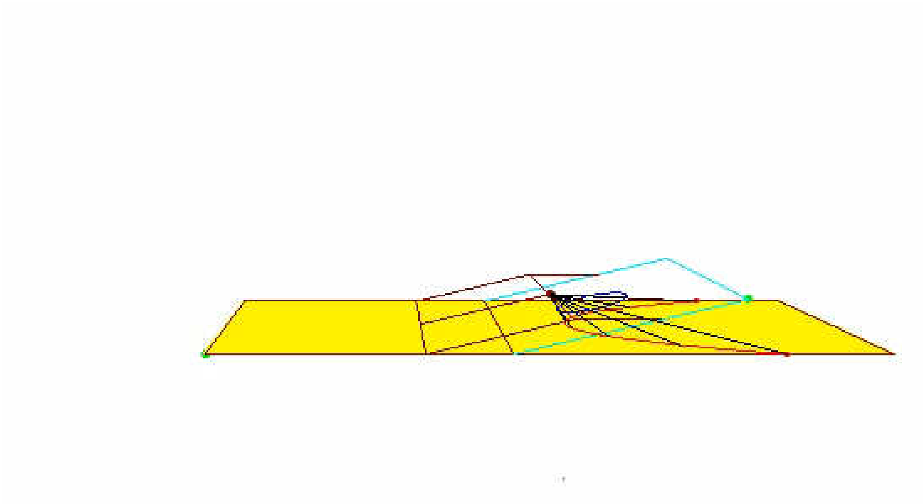




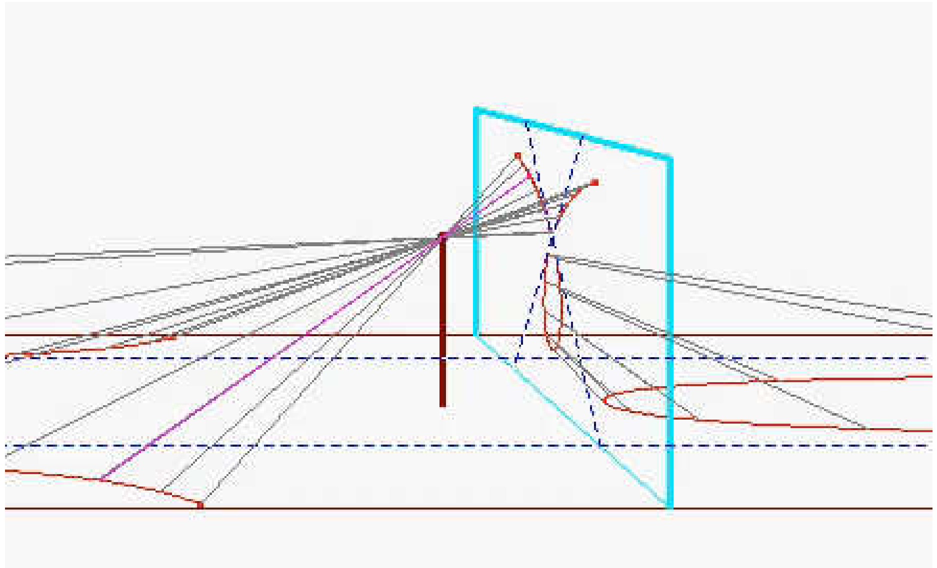


(2) Drawing by Leonardo in Manoscritto C, Bibliothèque de l'Institut de France, Parigi, fol.9)
(3) Cf. G. Bauer, Experimental Shadow Casting and the Early History of Perspective, in The Art Bulletin, June 1987 Vol. LXIX Number 2.
(4) Cf. A. De Rosa, Geometrie dell'ombra (Storia e simbolismo della teoria delle ombre), CittàStudiEdizioni, Milano 1997.
(5) Cf. lettera di Galilei a Grienberger, sett. 1611; Pietro Accolti, Lo inganno degli occhi, Firenze 1625. More information in F. Camerota, L'occhio e la lente, in Il segno di Masaccio, catalogo Giunti, Firenze 2001 (7) F. D'Aguillon, Opticorum libri sex, Anversa 1613, Libro VI.
(8) Cf. A. De Rosa, op. cit.
(9) G. Frisius, De Astrolabo Catholico (Anversa 1533). Cf. R. Sinisgalli, op.cit. in (11).
(10) G. Del Monte, Planisphaeriorum universalium theorica, Pesaro 1579. Cf. R. Sinisgalli,.op. cit. in (11).
(11) Cf. R. Sinisgalli, Gli studi di F. Commandino sul planisfero tolemaico come elemento di rottura nella tradizione della teoria prospettica della Rinascenza, Atti del Convegno di Milano 11-15 ottobre 1977, a cura di Dalai Emiliani, Firenze 1980 and F. Camerota, L'immagine del cielo, Arte Militare, Nuova maniera di levar piante, in Nel segno di Masaccio, catalogo Giunti, Firenze 2001. Moreover, cf. M. Kemp, La Scienza dell'Arte, Giunti 1990, Cap. IV, pag. 187 e segg. Docci - Maestri, Storia del rilevamento architettonico e urbano, Roma - Bari 1993, pag. 115
(12) F. Commandino, Ptolomaei Planisphaerium, Jordani Planisphaerium, Federici Commandini Urbinatis in Planisphaerium commentarius, in quo universa Scenografices ratio quam brevissime traditur, ac demonstratio-nibus confirmatur, Venezia 1558.
(13) Cf. M. Kemp, La scienza dell'arte, Giunti 1994, pagg. 99-101.
(14) F. d'Aguilon, Opticorum Libri VI, Philosophis juxta ac Mathematicis utiles, Anversa 1613.
(18) (Samuel van Hoogstraten, Inleyding tot de hooge schoole der schilderkonst, Rotterdam 1678).
(16) Both anamorphoses are obtained by making the light rays go through a cartboard frame with holes. In the first one they use a cylinder (M. Bettini, Apiaria Universae Philosophiae Mathematicae, Bologna 1641, 1645); in the second one they use a flat frame (J. Oznam, La Perspective Theorique et Pratique….., in Reécreations Mathématiques et Pysiques, Amsterdam 1683).
(17) Drawing by Rubens (Libro VI of the treatise by F.d'Aguilon, Opticorum Libri VI, 1613)

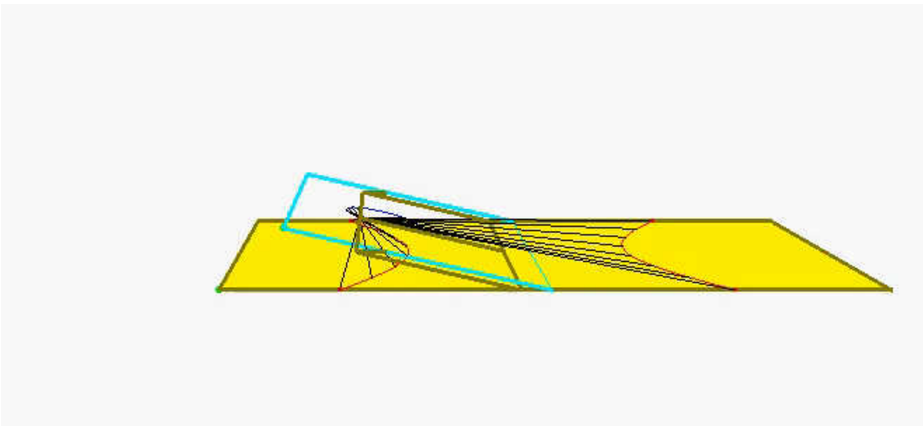
Projection of Conics Ellipse



Projection of Conic Nodes



Projection of Conics Hyperbola



Projection of Conics Parabola

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Vanishing Points

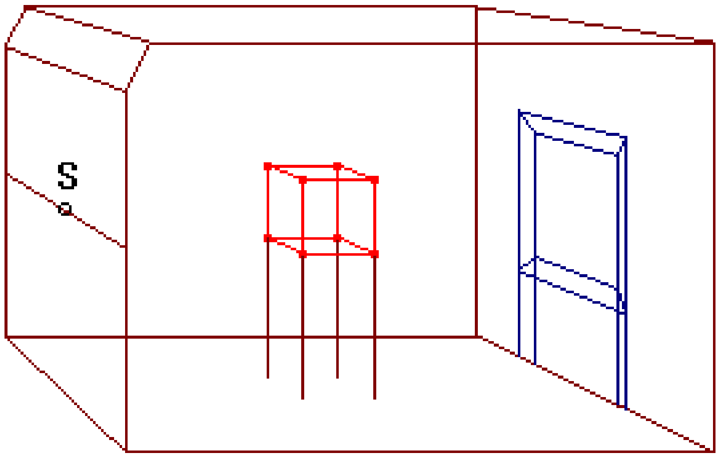
The cube in the box is identical to the smallest cube used in model 1. If the light source is positioned centrally in S, the shadow of the cube projected on the back wall coincides with the figure drawn on the transparent plate in the two previous models. It can be observed that:

1. When the light source is central (positioned in S), the shadow of the face of the cube parallel to the back wall and further from the light source is the internal square; while the shadow of the face of the cube parallel to the back wall and closer to the light source is the external square (with bigger sides). (cf. model 2) (Leibniz said that the shadow is a "reverse perspective").

2. The coincidence between shadow and perspective image (in this particular case) is easily observable, provided that in the projection of the shadow no parts that are essential to recognise the object are hidden by the opacity of the object itself:

this is unlikely to happen if reticular structures representing geometric schema are used (as in this example).

3. When observing shadows the viewer does not need to be in a fixed place (as it is sometimes necessary in the case of perspective). He can be in any position with respect to the light source: on top, at the bottom, on the side. In this model, the light source can be moved to the left or to the right of position S, generating a continuum of different shadows for the cube. The configurations so obtained are also perspectives: if we looked from the position where the light source is, each image would superimpose to the cube identical to the one in model 2. However, while in model 2 the different perspectives would need to be drawn one by one according to the different positions of the eye, in this model they are all produced automatically during the motion of the light source.



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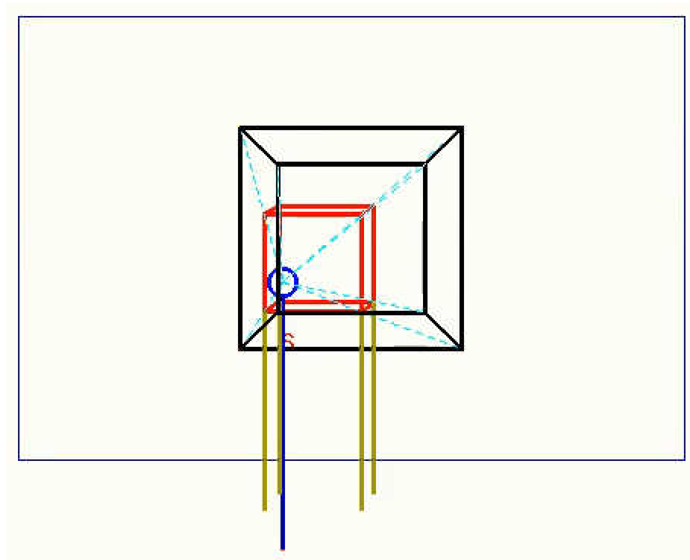
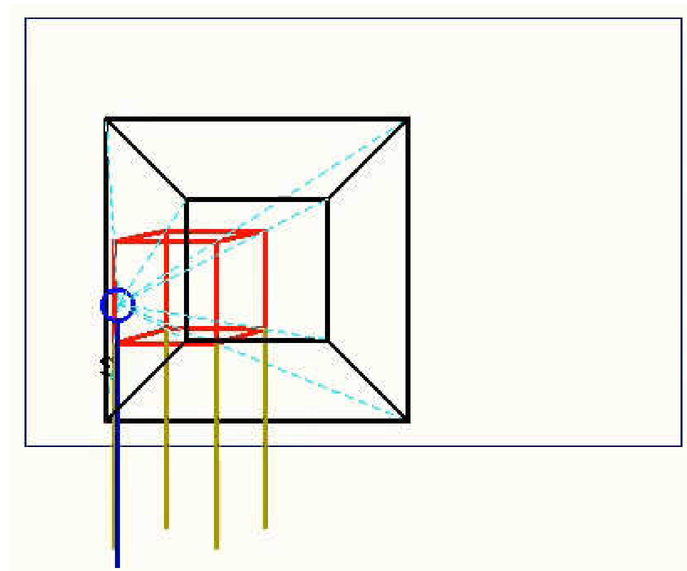
Vanishing Points 1



Vanishing Points 2



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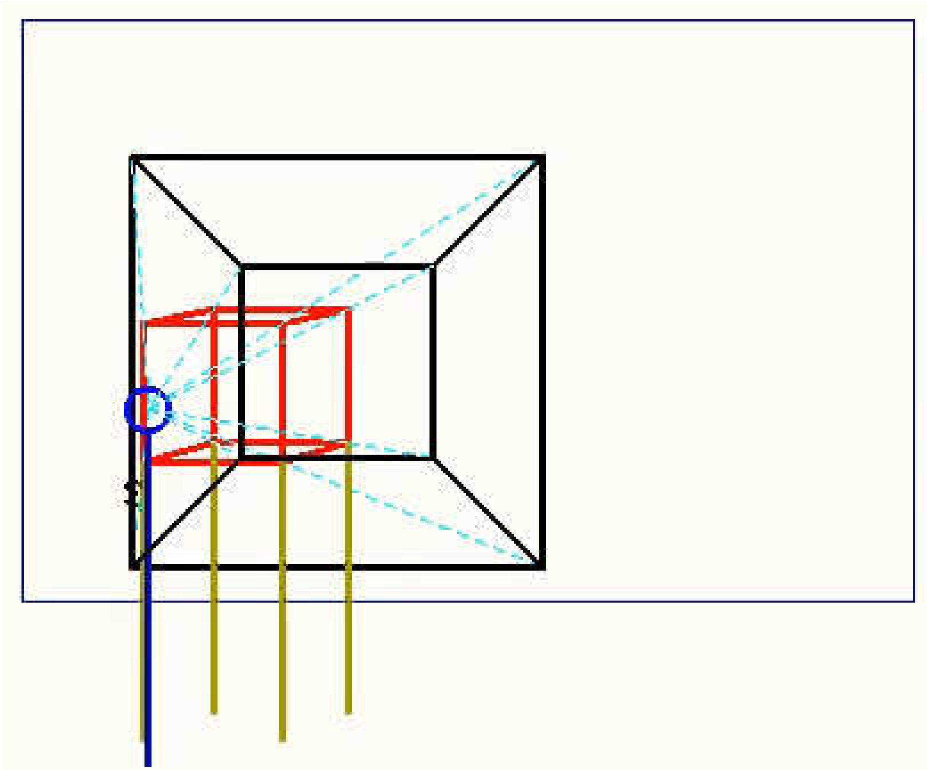
Vanishing Points 3

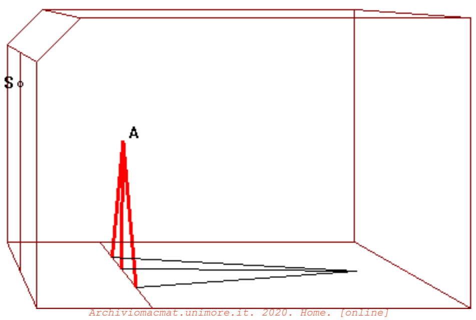


This model (together with models 5 and 6) verifies a property that was implicitely taken for granted in the description of the previous three models: the perpsective (shadow) of a line is a line. That is: central projections preserve linearity.

Moreover: Left-hand side

The brass rods joints together in A represent lines belonging to a vertical plane and meeting at A. A light source S, external to the plane, projects their shadows on a horizontal plane. When the distance of S and A from the ground is the same (and only in this case), then the shadows are parallel lines that meet the ones

projected on the intersection line between the vertical and the horizontal plane. This property can be verified by moving the light source up and down. To observe: in order to see the parallelism you need to focus on the external or internal contour of the projected shadows; the borders of each single rod diverge.



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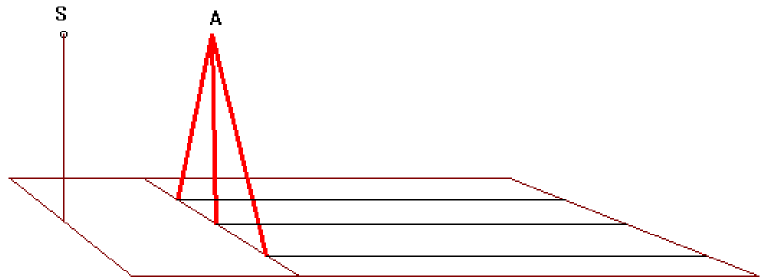
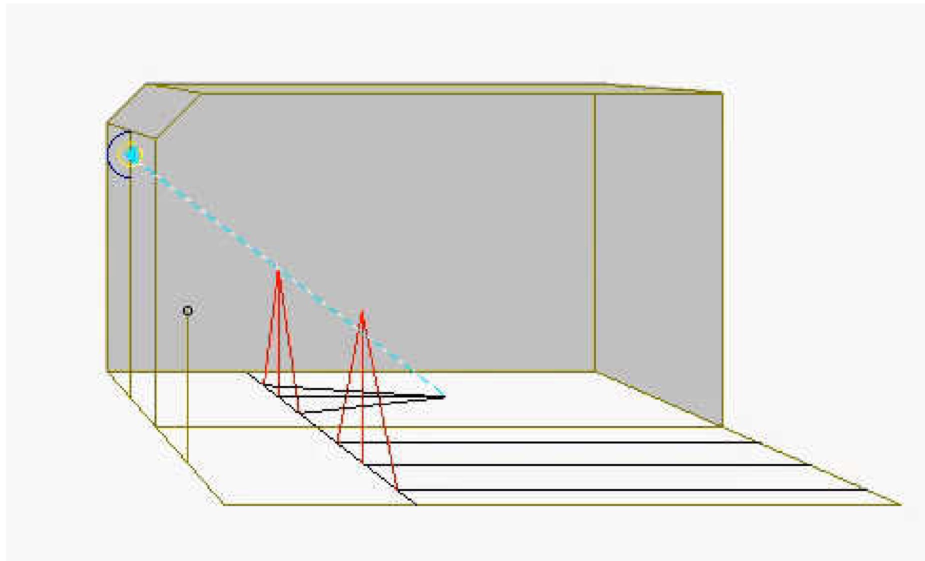
Right-hand side:

The perspective image on a vertical plane of parallel lines, lying on the horizontal plane and observed by a viewer whose eye is fixed in S, is constituted by lines meeting at a point A. Point A and the viewer's eye are at the same distance from the ground. Moreover, the points of intersection between the image lines and the parallel lines belong to the intersection line between the frame and the ground plane. The (converging) lines on the vertical plane could be linked to the (parallel) ones on the horizontal plane through straight strings going through S. In the same way as in model 1, these strings may represent either visual rays going to the eye or light rays coming from a ligth source.





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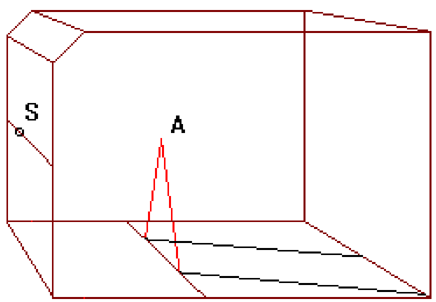
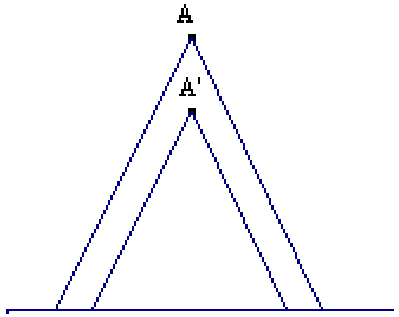
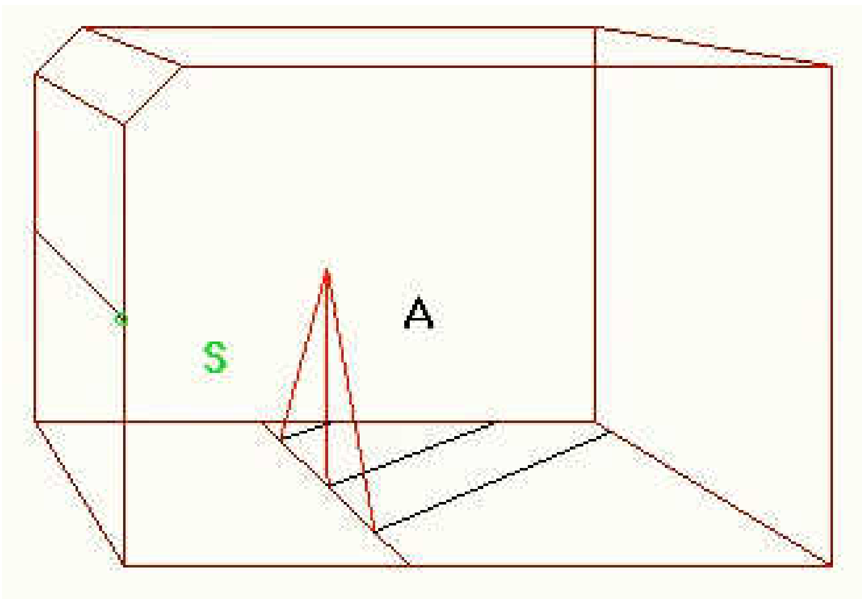
The situation presented here is similar to model 4.In this model, the light source can be moved to the right and to the left of S, on a line parallel to the ground and at the same distance from the ground as point A (that is the point where the brass rods representing the projected lines meet). The external borders of the shadows always stay parallel and they only change direction. In addition, the shadows and the brass rods meet where the rods intersect the horizontal plane (ground). To observe: the figure on the right shows the position of point A. The width of the rectangular convergent stripes represents the diameter of the brass cylindrical rods. If the height of the light source was the same as the height of the internal triangle (A'), the internal borders of the shadows would be parallel.





Second interpretation:
A viewer observes from a certain position parallel lines lying in the horizontal plane. Their perspective images on a vertical plane in front of the viewer's eye are lines converging at a point. When the lines observed change direction (but remaining parallel to each other), this point moves along a line on the vertical plane, parallel to the ground and at a distance from the ground equal to the distance of the viewer's eye from the ground (horizon line).



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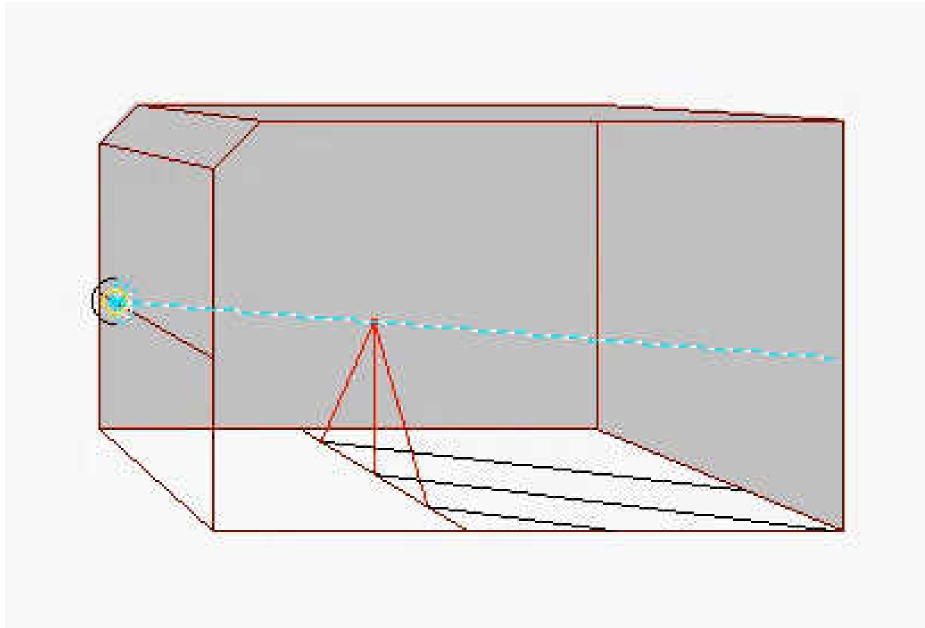

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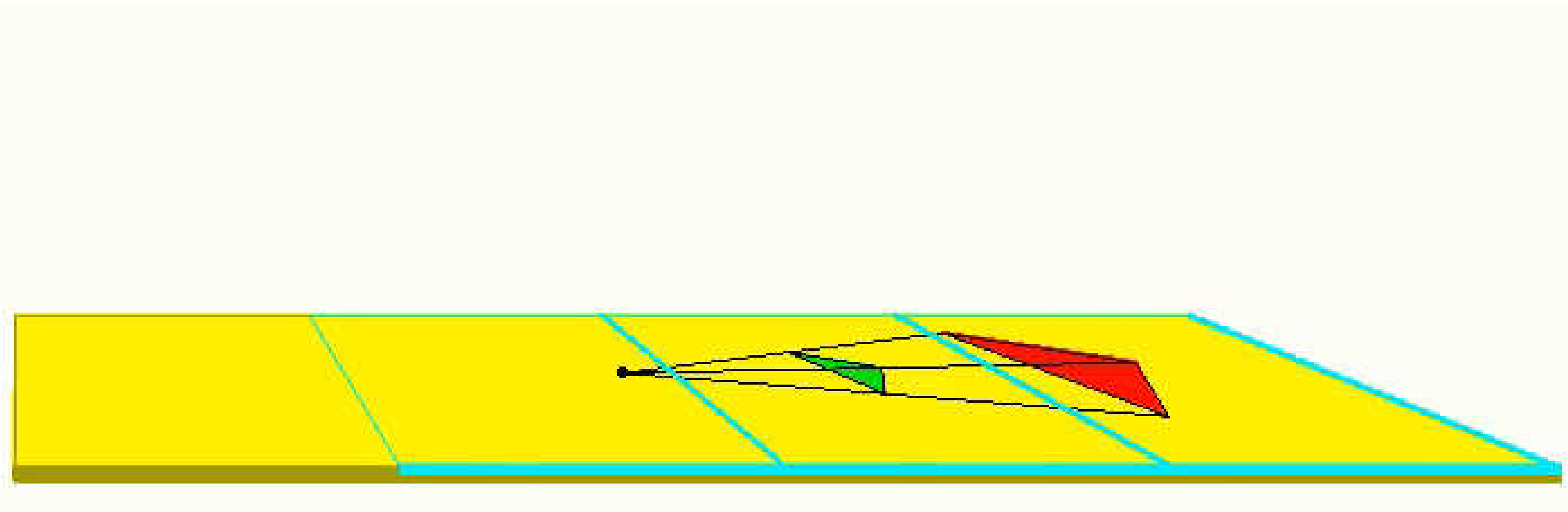
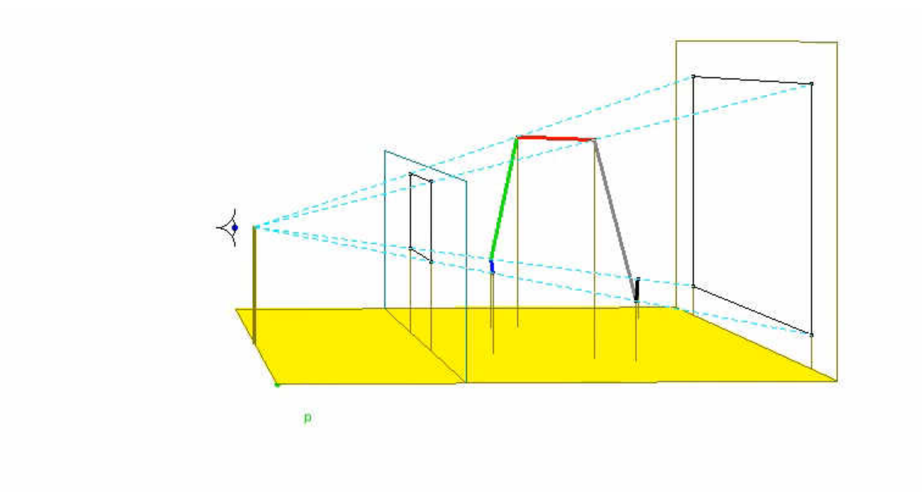
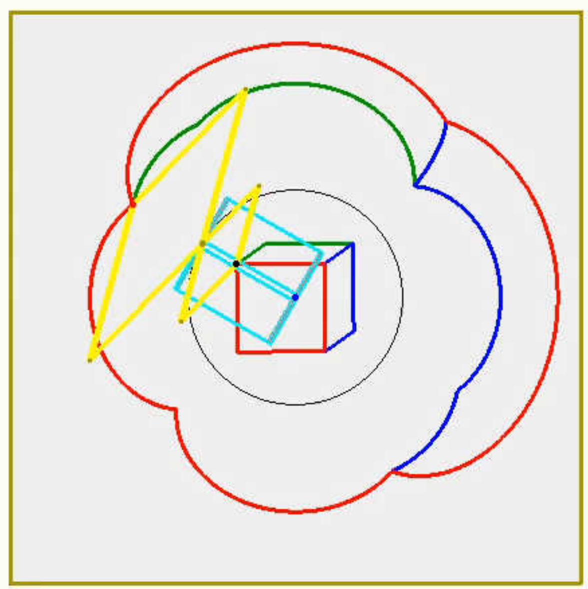
Stevin's Theorem



Different Objects with Same Perspective Image



Parré's Pantograph



Projection & Pantographs

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